



Law of INDICES

$$\Rightarrow * a \times a \times a \times \dots \dots \text{ n times } = a^n$$

$$* a^m \times a^n \times a^p = a^{m+n+p} \quad (a \neq 0)$$

$$* \frac{a^m}{a^n} = a^{m-n} \quad (m > n)$$

$$= \frac{1}{a^{n-m}} \quad (n > m)$$

$$= 1 \quad (m=n)$$

$$* (a^m)^n = a^{m \times n} = a^{n \times m} = (a^n)^m$$

$$* (abc)^n = a^n \times b^n \times c^n$$

$$* \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$$

$$* a^{\frac{p}{q}} = a^{\frac{1}{q} \times p} = (a^{\frac{1}{q}})^p = (a^p)^{\frac{1}{q}}$$

$$* \text{ if } a^m = a^n \text{ then } m=n$$

$$\text{if } a^m = b^m \text{ then } a=b$$

$$\begin{aligned} \# (a^m)^n &\neq a^{m^n} \\ (3^2)^4 &= 3^8 \neq 3^{16} \end{aligned}$$





* $a^0 = 1$

* $a^{-1} = \frac{1}{a}$ ($a \neq 0$)

* $a^{-n} = \frac{1}{a^n}$ & $a^n = \frac{1}{a^{-n}}$

* $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

* $(-1)^n = +1$ ($n = \text{even}$)
 $= -1$ ($n = \text{odd}$)

Objective Type Questions

With Solutions...

1. If $(3^{33} + 3^{33} + 3^{33})(2^{33} + 2^{33}) = 6x$, then, what is the value of x ?

- 1) 34 2) 35 3) 33 4) 33.5

$$3 \times 3^{33} \times 2 \times 2^{33} \rightarrow 3^{34} \times 2^{34} \rightarrow 6^{34} = 6x$$

$\therefore \boxed{x = 34}$

2. If $8^{3x-5} = \frac{1}{32^{7-4x}}$, then $x = ?$

- 1) $\frac{16}{9}$ 2) $\frac{20}{11}$ 3) $\frac{25}{13}$ 4) 2

$$(2^3)^{3x-5} = \frac{1}{(2^5)^{7-4x}} \rightarrow 2^{9x-15} = \frac{1}{2^{35-20x}}$$

$$\therefore 2^{9x-15} = 2^{20x-35} \quad \therefore 9x-15 = 20x-35$$

$\boxed{x = \frac{20}{11}}$

3. If $625^{3x-3} = 25^{6^{148}}$, then $x = ?$

- 1) 2 2) 3 3) 4 4) 5

$$1^{148} = 1 \quad \therefore 25^{4x-6} = 25^6 \quad \therefore 4x-6 = 6$$

$\boxed{x=3}$

4. If $2^{4x-1} = 4^x$, then find the value of 16^x .

- 1) 3 2) 2 3) 4 4) 5

$$2^{4x-1} = 2^{2x} \quad \therefore 4x-1 = 2x \quad \therefore 2x = 1 \quad \therefore x = \frac{1}{2}$$

$$\therefore 16^x \Rightarrow 2^{4x} \Rightarrow 2^{4 \times \frac{1}{2}} \Rightarrow 2^2 \Rightarrow 4$$

5. If $2^{x+y-2z} = 8^{8z-5-y}$, $5^{4y-6z} = 25^{y+z}$, $3^{4x-3y} = 9^{x+z}$ then the value of $2x+3y+5z$ is:

1) 56

2) 44

3) 32

4) 28

$$\begin{aligned} 2^{x+y-2z} &= 2^{3(8z-5-y)} & 5^{4y-6z} &= 5^{2(y+z)} & 3^{4x-3y} &= 3^{2(x+z)} \\ \therefore x+y-2z &= 24z-15-3y & 4y-6z &= 2y+2z & 4x-3y &= 2x+2z \\ \therefore x+4y-26z &= -15 & \therefore y = 4z & & \therefore 2x = 5z & \\ \left(\begin{array}{l} \frac{5}{2}z + 16z - 26z = -15 \\ \frac{-15}{2}z = -15 \\ z = 2 \end{array} \right) & & \downarrow & & x = \frac{5}{2}z & \\ & & y = 8 & & & \\ & & & & & \\ & & & & x = \frac{5}{2} \times 2 & \\ & & & & x = 5 & \end{aligned}$$

$$\therefore 2x+3y+5z = 10+24+10 = 44$$

6. If $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$, what is the value of a?

1) -6

2) -5

3) -7

4) -8

$$\therefore 3-5a = 17-3a \quad \therefore a = -7$$

7. If $(x^x)^{\frac{5}{4}} = x^{\frac{5}{4}}$, then x equal to?

1) $\frac{125}{64}$

2) $\frac{625}{256}$

3) $\frac{25}{16}$

4) $\frac{5}{4}$

$$\begin{aligned} x^{\frac{5}{4}x} &= x^{x\frac{5}{4}} \quad \therefore \frac{5}{4}x = x^{\frac{5}{4}} \\ \rightarrow \frac{5}{4}x &= x^{\frac{1}{4}} \times x^{\frac{1}{4}} \quad \rightarrow x^{\frac{1}{4}} = \frac{5}{4} \quad \rightarrow (x^{\frac{1}{4}})^4 = (\frac{5}{4})^4 \\ \rightarrow x &= \frac{625}{256} \end{aligned}$$

8. If $x^{x\sqrt{x}} = (x\sqrt{x})^x$, then x equal to

1) 4/9

2) 16/9

3) 3/2

4) 9/4



$$x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$$

$$\textcircled{x}^{\frac{3}{2}} = (x^{\frac{3}{2}})^x = \textcircled{x}^{\frac{3}{2}x} \therefore x^{\frac{3}{2}} = \frac{3}{2}x$$

$$\hookrightarrow x^1 \times x^{\frac{1}{2}} = \frac{3}{2}x \quad \hookrightarrow \sqrt{x} = \frac{3}{2} \quad \therefore x = \boxed{\frac{9}{4}}$$

9. If $3x + y = 81$ and $81x - y = 3$, then $x \times y = ?$

1) $\frac{255}{64}$

2) $\frac{125}{12}$

3) $\frac{240}{64}$

4) None

$$3^{x+y} = 3^4 \quad \& \quad 3^{4(x-y)} = 3^1$$

$$\therefore x+y = 4 \quad \therefore x = \frac{17}{8}, y = \frac{15}{8}$$

$$x-y = \frac{1}{4} \quad xy = \frac{17}{8} \times \frac{15}{8} = \boxed{\frac{255}{64}}$$

10. If $3^{8^{2x}} = 81^{2^{5x}}$ then $x = ?$

$$3^{8^{2x}} = 81^{2^{5x}} \rightarrow 3^{8^{2x}} = (3^4)^{2^{5x}} \rightarrow 3^{8^{2x}} = 3^{4 \times 2^{5x}}$$

$$3^{8^{2x}} = 3^{2^{2 \times 5x}} \rightarrow 3^{8^{2x}} = 3^{2^{5x+2}} \therefore 8^{2x} = 2^{5x+2}$$

$$\rightarrow 2^{6x} = 2^{5x+2} \quad \therefore 6x = 5x+2 \quad \therefore \boxed{x=2}$$

11. If $\frac{3^{a+3} \times 4^{a+6} \times 25^{a+1}}{27^{a-1} \times 8^{a-2} \times 125^{a+4}} = \frac{4}{15^{26}}$ then the value of $\sqrt{a+9}$ is:

$$\frac{4^{a+6}}{8^{a-2}} = 4 \rightarrow \frac{2^{2a+12}}{2^{3a-6}} = 4 \rightarrow 2^{-a+18} = 2^2$$

$$\therefore -a+18=2 \quad \therefore a=16 \quad \therefore \sqrt{16+9} = \boxed{5}$$

12. $\left[\left\{ \left(\frac{2}{3} \right)^3 \right\}^{(2x+3)} \right]^{-\frac{3}{4}} = \left[\left\{ \left(\frac{2}{3} \right)^{\frac{2}{3}} \right\}^{(3x+7)} \right]^{-\frac{6}{5}}$, then the value of $\sqrt{2-42x}$ is :

1) 5

2) 6

3) 3

4) 4

$$3 \times (2x+3) \left(-\frac{3}{4} \right) = \frac{2}{3} \times (3x+7) \left(-\frac{6}{5} \right)^2$$

$$\rightarrow 9x+135 = 48x+112 \quad \therefore 42x = -23 \quad \therefore \sqrt{2+23} = \boxed{5}$$



13. If x and y are natural number such that $x + y = 2021$, then what is the value of $(-1)^x + (-1)^y$?

x మరియు y లు సహజ సంఖ్యలు $x + y = 2021$. అయిన $(-1)^x + (-1)^y = ?$

(a) 2

(b) -2

(c) 0

(d) 1

$$\begin{array}{l} \downarrow \\ \text{even} \end{array} \quad \begin{array}{l} \downarrow \\ \text{odd} \end{array} \quad \begin{array}{l} \downarrow \\ \text{odd} \end{array} \quad \therefore (-1)^x + (-1)^y \\ = +1 - 1 = \boxed{0}$$

14. If $\frac{9^n \times 3^2 \times \left(3^{\frac{n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{729}$, then $m-n = ?$

1) 3

2) 1

3) 2

4) -2

$$\begin{aligned} \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} &\Rightarrow \frac{3^{3n} \times 3^2 - 3^{3n}}{3^{3m} \times 2^3} \\ \rightarrow \frac{3^{3n} (3^2 - 1)}{3^{3m} \times 2^3} &\rightarrow \frac{1}{3^{3m-3n}} = \frac{1}{3^6} \\ \therefore 3m - 3n = 6 &\quad \therefore \boxed{m-n=2} \end{aligned}$$

15. The value of $\frac{(81)^{3.6} \times (9)^{2.7}}{(81)^{4.2} \times (3)}$ is

1) 3

2) 6

3) 9

4) 1

$$\frac{1}{81^{0.6}} \times \frac{3^{5.4}}{3} \rightarrow \frac{1}{3^{2.4}} \times \frac{3^{5.4}}{3^1} \rightarrow \frac{3^{5.4}}{3^{3.4}} \rightarrow 3^2 = \boxed{9}$$

16. Find the square root of

1) $7+4\sqrt{3}$

2) $4+\sqrt{15}$

3) $\sqrt{61+28\sqrt{3}}$

4) $139-80\sqrt{3}$

5) $74-12\sqrt{30}$

$$(A) \sqrt{7+4\sqrt{3}} = (2+\sqrt{3})$$

$$4\sqrt{3} \rightarrow \textcircled{2} \times \frac{2\sqrt{3}}{\sqrt{3}} \\ \sqrt{4} + \sqrt{3} = \textcircled{1}$$

$$(B) \sqrt{4+\sqrt{15}} \rightarrow \sqrt{\frac{8+2\sqrt{15}}{2}} \\ = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$$

$$\frac{2\sqrt{15}}{\sqrt{5} + \sqrt{3}}$$

$$(C) \sqrt{61+28\sqrt{3}} = (7+2\sqrt{3})$$

$$28\sqrt{3} \rightarrow \textcircled{2} \times \frac{14\sqrt{3}}{2\sqrt{3}}$$



$$(D) \sqrt{139 - 80\sqrt{3}} = (5\sqrt{3} - 8)$$

$$80\sqrt{3} \rightarrow ② \times 5 \times 8 \times \sqrt{3}$$

$$\begin{matrix} 8^2 \\ 64 + \end{matrix} \quad \begin{matrix} (\sqrt{3})^2 \\ 75 \end{matrix}$$

$$(E) \sqrt{74 - 12\sqrt{30}} = (3\sqrt{6} - 2\sqrt{5})$$

$$12\sqrt{30} \rightarrow ② \times 2 \times 3 \times \sqrt{5} \times \sqrt{6}$$

$$\begin{matrix} 2\sqrt{5} \\ 20 + \end{matrix} \quad \begin{matrix} 3\sqrt{6} \\ 54 = 74 \end{matrix}$$

$$17. \sqrt{6 - \sqrt{35}} = ?$$

$$1) \frac{1}{\sqrt{2}}(\sqrt{7} - \sqrt{5})$$

$$2) \frac{1}{\sqrt{2}}(\sqrt{5} - \sqrt{7})$$

$$3) \frac{1}{4}(\sqrt{7} - \sqrt{5})$$

$$4) \frac{1}{4}(\sqrt{7} + \sqrt{3})$$

$$\sqrt{6 - \sqrt{35}} = \sqrt{\frac{12 - 2\sqrt{35}}{2}}$$

$$2\sqrt{35} \rightarrow ② \times \sqrt{5} \times \sqrt{7}$$

$$\hookrightarrow \boxed{\frac{\sqrt{7} - \sqrt{5}}{\sqrt{2}}}$$

$$18. \text{ The value of } \frac{1}{\sqrt{7} - 4\sqrt{3}} \text{ is closet to:}$$

$$1) 1.2$$

$$2) 4.1$$

$$3) 4.2$$

$$4) 3.7$$

$$\frac{1}{\sqrt{7}-4\sqrt{3}} = \frac{1}{2-\sqrt{3}} \rightarrow 2+\sqrt{3} \rightarrow 2+1.732 \rightarrow \boxed{3.7}$$

$$19. \text{ The value of } \frac{1}{\sqrt{17} + 12\sqrt{2}} \text{ is closet to....}$$

$$1) 1.4$$

$$2) 1.2$$

$$3) 0.14$$

$$4) 0.17$$

$$\frac{1}{\sqrt{17}+12\sqrt{2}} = \frac{1}{3+2\sqrt{2}}$$

$$\hookrightarrow 3 - 2\sqrt{2} = 3 - 2.82 \approx \boxed{0.17}$$

$$12\sqrt{2} \rightarrow ② \times 6\sqrt{2}$$

$$\begin{matrix} 3 \\ 9 \end{matrix} \quad \begin{matrix} 2\sqrt{2} \\ 8 \end{matrix}$$

$$20. \text{ The value of } \frac{14}{\sqrt{43} + 30\sqrt{2}} \text{ is closet to:}$$

$$1) 1.762$$

$$2) 1.414$$

$$3) 1.823$$

$$4) 1.516$$



$$\sqrt{43+30\sqrt{2}} = (5 + 3\sqrt{2})$$

$$\therefore \frac{14}{5+3\sqrt{2}} = \frac{2+4(5-3\sqrt{2})}{\cancel{25}} \rightarrow 30\sqrt{2} \rightarrow 2 \times 3 \times 5 \times \sqrt{2}$$

$$\hookrightarrow 10 - 6\sqrt{2} = 10 - 8.484 = \boxed{1.516}$$

$$\begin{array}{c} 5^2 \\ 25 \end{array} \quad \begin{array}{c} (3\sqrt{2})^2 \\ 18 \end{array}$$

21. The value of $\sqrt{9-2\sqrt{11-6\sqrt{2}}}$ is closest to:

- 1) 2.7 2) 2.9 3) 2.4 4) 2.1

$$\sqrt{11-6\sqrt{2}} = (3-\sqrt{2})$$

$$\sqrt{9-2(3-\sqrt{2})} = \sqrt{3+2\sqrt{2}}$$

$$6\sqrt{2} = 2 \times 3 \times \sqrt{2}$$

$$3^2 + (\sqrt{2})^2 = 11$$

$$\hookrightarrow (\sqrt{2}+1) = \boxed{2.4}$$

22. If $\sqrt{10-2\sqrt{21}} + \sqrt{8+2\sqrt{15}} = \sqrt{a} + \sqrt{b}$, where a and b are positive integers, then the value of \sqrt{ab} is closest to

- 1) 5.9 2) 6.8 3) 4.6 4) 7.2

$$\sqrt{10-2\sqrt{21}} = (\sqrt{7}-\sqrt{3}), \quad \sqrt{8+2\sqrt{15}} = (\sqrt{5}+\sqrt{3})$$

$$\therefore \sqrt{7}-\sqrt{3} + \sqrt{5} + \sqrt{3} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{7} + \sqrt{5} = \sqrt{a} + \sqrt{b} \quad \therefore \sqrt{ab} = \sqrt{35} \approx \boxed{5.9}$$

23. The value of $\sqrt{28+10\sqrt{3}} - \sqrt{7-4\sqrt{3}}$ is closest to:

- 1) 5.8 2) 6.5 3) 6.1 4) 7.2

$$5+\sqrt{3} - (2-\sqrt{3}) \rightarrow 3+2\sqrt{3} \rightarrow 3+3 \cdot 4 = \boxed{16.5}$$

24. If $\sqrt{11-3\sqrt{8}} = a+b\sqrt{2}$, then what is the value of $(2a+3b)$?

- 1) 5 2) 7 3) 9 4) 3

$$\sqrt{11-3\sqrt{8}} = 3-\sqrt{2}$$

$$3-\sqrt{2} = a+b\sqrt{2}$$

$$3\sqrt{8} = 3 \times 2\sqrt{2} = 2 \times 3 \times \sqrt{2}$$

$$3 \quad \sqrt{2}$$

$$a=3, b=-1 \quad \therefore (2a+3b) = 6-3 = \boxed{3}$$

25. If $\sqrt{86-60\sqrt{2}} = a-b\sqrt{2}$, then what will be the value of $\sqrt{a^2+b^2}$, correct to one decimal place?

- 1) 8.4 2) 7.8 3) 8.2 4) 7.2

$$\sqrt{86-60\sqrt{2}} = 5\sqrt{2}-6$$

$$60\sqrt{2} = 2 \times 5 \times 6 \times \sqrt{2}$$

$$6^2 \quad (5\sqrt{2})^2$$

$$36 + \frac{50}{50} = 86$$

$$\therefore 5\sqrt{2}-6 = a-b\sqrt{2}$$



$$a = -6 \quad b = -5 \quad \sqrt{a^2+b^2} = \sqrt{36+25} = \sqrt{61} \approx \boxed{7.8}$$

26. If $x = \sqrt{-\sqrt{3}} + \sqrt{3+8\sqrt{7+4\sqrt{3}}}$, where $x > 0$, then the value of x is equal to:

- 1) 2 2) 3 3) 4 4) 1

$$\sqrt{-\sqrt{3}} = 2 + \sqrt{3} \quad \therefore \quad \sqrt{3+8(2+\sqrt{3})} = \sqrt{19+8\sqrt{3}}$$

$$\rightarrow 4 + \sqrt{3} \rightarrow \sqrt{-\cancel{\sqrt{3}} + 4 + \sqrt{3}} = \boxed{2}$$

27. The value of $\frac{1}{(9-4\sqrt{5})^2} + \frac{1}{(9+4\sqrt{5})^2}$ is:

- 1) 322 2) 424 3) 246 4) 286

$$\frac{(9+4\sqrt{5})^2 + (9-4\sqrt{5})^2}{4} \Rightarrow (a+b)^2 + (a-b)^2 = 2(a^2+b^2)$$

$$\hookrightarrow 2(81+80) = \boxed{322} \Rightarrow (a+b)^2 - (a-b)^2 = 4ab$$

28. $\sqrt{7+4\sqrt{3}} - \sqrt{28+10\sqrt{3}} + \frac{\sqrt{11}}{\sqrt{20+6\sqrt{11}} + \sqrt{20-6\sqrt{11}}} = ?$

- 1) $2\frac{1}{2}$ 2) $-2\frac{1}{2}$ 3) $3\frac{1}{2}$ 4) $-3\frac{1}{2}$

$$2 + \sqrt{3} - (5 + \sqrt{3}) + \frac{\sqrt{11}}{3 + \sqrt{11} + (\sqrt{11} - 3)}$$

$$\Rightarrow -3 + \frac{1}{2} \Rightarrow \boxed{-2\frac{1}{2}}$$

29. $\frac{6^2 + 7^2 + 8^2 + 9^2 + 10^2}{\sqrt{7+4\sqrt{3}} - \sqrt{4+2\sqrt{3}}} = ?$

- 1) 330 2) 305 3) 355 4) 366

$$\frac{330}{2 + \sqrt{3} - (\sqrt{3} + 1)} = \frac{330}{1} = \boxed{330}$$

30. If $\sqrt{15} + \sqrt{60} + \sqrt{84} + \sqrt{140} = \sqrt{a} + \sqrt{b} + \sqrt{c}$, then the value of $a + b + c$?

- 1) 5 2) 20 3) 10 4) 15

$$\begin{array}{cccc} 15 & + & \sqrt{60} & + \sqrt{84} + \sqrt{140} \\ \downarrow & & \downarrow & \downarrow \\ a+b+c & 2 \times \sqrt{5} \times \sqrt{3} & 2 \times \sqrt{3} \times \sqrt{7} & 2 \times \sqrt{7} \times \sqrt{5} \end{array}$$



$$\therefore \sqrt{a} + \sqrt{b} + \sqrt{c} = \sqrt{5} + \sqrt{3} + \sqrt{7}$$

$$a+b+c = 15 \quad \& \quad abc = 5 \times 3 \times 7 = 105$$

31. The expression $\sqrt{10+2(\sqrt{6}-\sqrt{15}-\sqrt{10})}$ is equal to:

- | | |
|----------------------------------|---------------------------------|
| 1) $\sqrt{3}-\sqrt{2}-\sqrt{5}$ | 2) $\sqrt{3}-\sqrt{2}+\sqrt{5}$ |
| 3) $\sqrt{2}-\sqrt{3}-\sqrt{15}$ | 4) $\sqrt{3}+\sqrt{2}-\sqrt{5}$ |

By option verification,

-ve sign in 2 places $-\sqrt{15} + -\sqrt{10}$

\therefore (a) & option(i) & (c) & option(iii) are wrong
(because in between 3 -ve signs will not be placed.)

$$\begin{aligned}\sqrt{15} &= \sqrt{5} \times \sqrt{3} \\ \sqrt{10} &= \sqrt{5} \times \sqrt{2}\end{aligned} \quad] \quad \sqrt{5} \text{ is common} \quad \therefore \sqrt{5} = -\text{ve}$$

but in option (ii) $\sqrt{5}$ is given +ve
so op(ii) ✗

\therefore op(iv) is correct $\boxed{\sqrt{3}+\sqrt{2}-\sqrt{5}}$



32. What is the square root of $23+4\sqrt{10}-10\sqrt{2}-8\sqrt{5}$?

- | | |
|-----------------------------------|------------------------------------|
| 1) $\sqrt{5}+\sqrt{10}-2\sqrt{2}$ | 2) $\sqrt{5}+2\sqrt{2}+10$ |
| 3) $\sqrt{5}-\sqrt{10}+2\sqrt{2}$ | 4) $2\sqrt{2}-\sqrt{15}+\sqrt{10}$ |

$$+\text{ve} = 4\sqrt{10} \quad (\text{b}) \times \rightarrow \text{all (+ve)}$$

$$(\text{a}) \text{ multiply (+ve)} \rightarrow \sqrt{5} \times \sqrt{10} = \sqrt{50} \neq 4\sqrt{10} \quad (\text{a} \times)$$

$$(\text{c}) \quad \sqrt{5} \times 2\sqrt{2} \rightarrow 2\sqrt{10} \times 2 \rightarrow 4\sqrt{10} = 4\sqrt{10} \quad (\text{c} \vee)$$

33. If $\sqrt{24+4\sqrt{21}-2\sqrt{35}-4\sqrt{15}} + \sqrt{21+8\sqrt{5}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$ then $a^2 + b^2 + c^2 = ?$

- | | | | |
|--------|--------|--------|--------|
| 1) 449 | 2) 330 | 3) 705 | 4) 593 |
|--------|--------|--------|--------|

$$\begin{array}{ccccccc}
 24 & + & 4\sqrt{21} & - & 2\sqrt{35} & - & 4\sqrt{15} \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 \textcircled{2} \times 2 \times \sqrt{7} \times \sqrt{3} & & \textcircled{2} \times \sqrt{7} \times \sqrt{5} & & \textcircled{2} \times 2 \times \sqrt{3} \times \sqrt{5} \\
 2\sqrt{3} & \swarrow & \searrow & & \sqrt{5} & \swarrow & \searrow \\
 & & \sqrt{7} & & & & 2\sqrt{3}
 \end{array}$$

$$\begin{aligned}
 &\hookrightarrow 2\sqrt{3} + \sqrt{7} - \cancel{\sqrt{5}} + (4 + \cancel{\sqrt{5}}) \\
 &\hookrightarrow 2\sqrt{3} + \sqrt{7} + 4 = \sqrt{a} + \sqrt{b} + \sqrt{c} \\
 &\hookrightarrow \sqrt{12} + \sqrt{7} + \sqrt{16} = \sqrt{a} + \sqrt{b} + \sqrt{c} \\
 \therefore a^2 + b^2 + c^2 &= 12^2 + 7^2 + 16^2 = \boxed{449}
 \end{aligned}$$

34. $\frac{(\sqrt{3} + \sqrt{5})(\sqrt{5} + \sqrt{2})}{\sqrt{2} + \sqrt{3} + \sqrt{5}} = ?$

1) $(\sqrt{2} + \sqrt{3} + \sqrt{5})$

2) $\frac{3}{2}(\sqrt{2} + \sqrt{3} + \sqrt{5})$

3) $\frac{1}{4}(\sqrt{2} + \sqrt{3} + \sqrt{5})$

4) $\frac{1}{2}(\sqrt{2} + \sqrt{3} + \sqrt{5})$

$$\begin{aligned}
 \sqrt{2} &\approx 1.4 \\
 \sqrt{3} &\approx 1.7 \\
 \sqrt{5} &\approx 2.2 \\
 (\text{Add}) \rightarrow \underline{\underline{5.3}}
 \end{aligned}$$

$$\frac{1.3 \cdot 1.7 \times 3.7}{-5 \cdot 3 \cdot 3}^2 \Rightarrow \approx 2.6$$

\therefore option (d) ≈ 2.6

35. If $x = 5 - \sqrt{21}$, then $\frac{\sqrt{x}}{\sqrt{32 - 2x - \sqrt{21}}} = ?$

1) $\frac{1}{2}(\sqrt{3} - \sqrt{7})$

2) $\frac{1}{\sqrt{2}}(7 + \sqrt{3})$

3) $\frac{1}{\sqrt{2}}(\sqrt{7} - \sqrt{3})$

4) $\frac{1}{\sqrt{2}}(\sqrt{7} + \sqrt{3})$

$$\frac{\sqrt{5 - \sqrt{21}}}{\sqrt{22 + 2\sqrt{21}} - \sqrt{21}} = \frac{\sqrt{5 - \sqrt{21}}}{\sqrt{21} + 1 - \sqrt{21}} = \sqrt{5 - \sqrt{21}}$$

$$\sqrt{\frac{10 - 2\sqrt{21}}{2}} = \frac{\sqrt{7} - \sqrt{3}}{\sqrt{2}}$$

36. $\frac{\sqrt{26 - 15\sqrt{3}}}{5\sqrt{2} - \sqrt{38 + 5\sqrt{3}}} = ?$

1) $\sqrt{2}$

2) $\frac{1}{\sqrt{3}}$

3) $\sqrt{3}$

4) $\frac{1}{\sqrt{2}}$



Multiply by $\sqrt{2}$ in Num & Den

$$\frac{\sqrt{52 - 2 \times 15\sqrt{3}}}{10 - \sqrt{16 + 2 \times 5\sqrt{3}}} \rightarrow \frac{3\sqrt{3} - 5}{10 - (5\sqrt{3} + 1)} \rightarrow \frac{(3\sqrt{3} - 5)}{\sqrt{3}(2\sqrt{3} - 5)} = \boxed{\frac{1}{\sqrt{3}}}$$

37. The expression $\frac{15(\sqrt{10} + \sqrt{5})}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}}$ is equal to:

- 1) $5 + 2\sqrt{2}$ 2) $5 - 2\sqrt{2}$ 3) $5(3 + 2\sqrt{2})$ 4) $10(3 + 2\sqrt{5})$

$$\begin{aligned} \frac{15(\sqrt{10} + \sqrt{5})}{\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}} &\rightarrow \frac{15(\sqrt{10} + \sqrt{5})}{3\sqrt{10} - 3\sqrt{5}} \\ \rightarrow \frac{5(\sqrt{10} + \sqrt{5})}{(\sqrt{10} - \sqrt{5})} &\rightarrow \cancel{5}(\sqrt{10} + \sqrt{5}) \cdot \frac{(\sqrt{10} + \sqrt{5})}{\cancel{5}} \rightarrow (\sqrt{10} + \sqrt{5})^2 \\ \rightarrow \boxed{5(3 + 2\sqrt{2})} \end{aligned}$$

38. $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}} = ?$, If $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

- 1) 5.498 2) 5.398 3) 6.398 4) 3.498

$$\begin{aligned} \frac{15}{\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}} &\rightarrow \frac{15}{3\sqrt{10} - 3\sqrt{5}} \rightarrow \frac{5}{\sqrt{10} - \sqrt{5}} \\ \hookrightarrow \cancel{5} \times \frac{\sqrt{10} + \sqrt{5}}{\cancel{5}} &\rightarrow \sqrt{10} + \sqrt{5} \rightarrow 3.162 + 2.236 = \boxed{5.398} \end{aligned}$$

39. Let $x = \sqrt[3]{27} - \sqrt{6\frac{3}{4}}$ and $y = \frac{\sqrt{45} + \sqrt{605} + \sqrt{245}}{\sqrt{80} + \sqrt{125}}$, then the value of $x^2 + y^2$ is:

- 1) $\frac{227}{9}$ 2) $\frac{221}{36}$ 3) $\frac{221}{9}$ 4) $\frac{223}{36}$

$$x = \sqrt{3} - \frac{3\sqrt{3}}{2} = \frac{-\sqrt{3}}{2}, \quad y = \frac{3\sqrt{5} + 11\sqrt{5} + 7\sqrt{5}}{4\sqrt{5} + 5\sqrt{5}} = \frac{1}{3}$$

$$x^2 + y^2 = \frac{3}{4} + \frac{49}{9} = \boxed{\frac{223}{36}}$$

40. $\frac{3\sqrt{7}}{\sqrt{5} + \sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2} + \sqrt{7}} + \frac{2 + \sqrt{2}}{\sqrt{7} + \sqrt{5}} = ?$

$$\frac{\cancel{3}\sqrt{7}(\sqrt{5} - \sqrt{2})}{\cancel{3}} - \frac{\cancel{5}\sqrt{5}(\sqrt{7} - \sqrt{2})}{\cancel{5}} + \frac{\cancel{2}\sqrt{2}(\sqrt{7} - \sqrt{5})}{\cancel{2}}$$

$$\hookrightarrow \cancel{\sqrt{35}} - \cancel{\sqrt{14}} - \cancel{\sqrt{35}} + \cancel{\sqrt{10}} + \cancel{\sqrt{14}} - \cancel{\sqrt{10}} = \boxed{0} \Rightarrow \text{Ans will be } 0 \text{ or in root.}$$



41. If $\frac{8+2\sqrt{3}}{3\sqrt{3}+5} = a\sqrt{3}-b$, then the value of $a+b$ is equal to:

- 1) 18 2) 15 3) 16 4) 24

$$\frac{(8+2\sqrt{3})(3\sqrt{3}-5)}{(3\sqrt{3}+5)(3\sqrt{3}-5)} \Rightarrow \frac{(8+2\sqrt{3})(3\sqrt{3}-5)}{2} \Rightarrow (4+\sqrt{3})(3\sqrt{3}-5)$$

$$\Rightarrow 7\sqrt{3}-11 = a\sqrt{3}-b \quad \therefore a+b = 7+11 = 18$$

42. $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}} = ?$

- 1) $10-\sqrt{99}$ 2) $\sqrt{2}-10$ 3) 7 4) 9

$$\frac{\cancel{\sqrt{2}}-1}{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cancel{\sqrt{5}} - \cancel{\sqrt{4}} + \dots + \cancel{\sqrt{100}} - \cancel{\sqrt{99}}$$

$$\therefore -1 + \sqrt{100} = -1 + 10 = \boxed{9}$$

43. $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} \dots + \frac{1}{\sqrt{2}-1} = ?$

- 1) 10 2) 9 3) 11 4) 12

$$\cancel{\sqrt{100}} + \cancel{\sqrt{99}} - \cancel{\sqrt{99}} - \cancel{\sqrt{98}} + \cancel{\sqrt{98}} + \cancel{\sqrt{97}} - \cancel{\sqrt{96}} \dots + \cancel{\sqrt{2}} + 1$$

$$\cancel{\sqrt{100}} + 1 = \boxed{11}$$

44. If $x = \sqrt{\left(\frac{5+2\sqrt{6}}{5-2\sqrt{6}}\right)}$, then $x^2(x-10)^2 = ?$

- 1) 1 2) -1 3) 2 4) -2

$$x = 5 + 2\sqrt{6}$$

$$x^2(x-10)^2$$

$$x-5 = 2\sqrt{6}$$

$$\Leftrightarrow [x(x-10)]^2$$

$$x^2-10x+25 = 24$$

$$\Leftrightarrow (x^2-10x)^2$$

$$x^2-10x = -1$$

$$\Leftrightarrow (-1)^2 = \boxed{1}$$

45. If x, y is a rational number and $\frac{5+\sqrt{11}}{3-2\sqrt{11}} = x + y\sqrt{11}$, then find the value of x and y ?

1) $x = \frac{-14}{17}, y = \frac{-13}{26}$

2) $x = \frac{4}{13}, y = \frac{11}{17}$

3) $x = \frac{-27}{25}, y = \frac{-11}{37}$

4) $x = \frac{-37}{35}, y = \frac{-13}{35}$



$$(5 + \sqrt{11}) \times \frac{(3 + 2\sqrt{11})}{-35} = x + y\sqrt{11} \Rightarrow \text{Ans -ve से होंगे}$$

वह den में 35 का factor होगा.

$$\hookrightarrow \frac{15 + 10\sqrt{11} + 3\sqrt{11} + 2^2}{-35} = x + y\sqrt{11}$$

$$\hookrightarrow \frac{37 + 13\sqrt{11}}{-35} = x + y\sqrt{11} \quad \therefore x = \boxed{\frac{-37}{35}}$$

$$y = \boxed{\frac{-13}{35}}$$

46. The value of $\frac{2\sqrt{10}}{\sqrt{5} + \sqrt{2} - \sqrt{7}} - \frac{\sqrt{5-2}}{\sqrt{5+2}} - \frac{3}{\sqrt{7}-2}$

1) $2\sqrt{5}$

2) $\sqrt{7}$

3) $2 + \sqrt{2}$

4) $\sqrt{2}$

$$\frac{2\sqrt{10}(\sqrt{5} + \sqrt{2} + \sqrt{7})}{(\sqrt{5} + \sqrt{2})^2 - (\sqrt{7})^2} - (\sqrt{5} - 2) - \frac{3(\sqrt{7} + 2)}{3}$$

$$\frac{2\sqrt{10}(\sqrt{5} + \sqrt{2} + \sqrt{7})}{2\sqrt{10}} - \sqrt{5} + \cancel{\sqrt{2}} - \sqrt{7} - \cancel{2}$$

$$\sqrt{5} + \sqrt{2} + \cancel{\sqrt{4}} - \cancel{\sqrt{5}} - \cancel{\sqrt{1}} = \boxed{\sqrt{2}}$$

47. The value of $5 - \frac{8+2\sqrt{15}}{4} - \frac{1}{8+2\sqrt{15}}$ is equal to:

1) $\frac{2}{3}$

2) 1

3) $\frac{1}{2}$

4) $\frac{1}{4}$

$$5 - \frac{8+2\sqrt{15}}{4} - \frac{(8-2\sqrt{15})}{4}$$

$$\hookrightarrow 5 - 2 - 2 = \boxed{1}$$

48. If $\frac{4}{1+\sqrt{2}+\sqrt{3}} = a+b\sqrt{2}+c\sqrt{3}-d\sqrt{6}$, where a, b, c, d are natural numbers, then the value of a + b + c + d.

1) 0

2) 2

3) 4

4) 1

$$\frac{4}{(1+\sqrt{2})+\sqrt{3}} \times \frac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} \rightarrow \frac{4(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2 - (\sqrt{3})^2} \rightarrow \frac{4(1+\sqrt{2}-\sqrt{3})}{3+2\sqrt{2}-3}$$

$$\rightarrow \sqrt{2}(1+\sqrt{2}-\sqrt{3}) \rightarrow \sqrt{2} + 2 - \sqrt{6} = a + b\sqrt{2} + c\sqrt{3} - d\sqrt{6}$$

$$\therefore a+b+c+d = 2+1+0+1 = \boxed{4}$$



49. The value of $5\sqrt{3} + 7\sqrt{2} - \sqrt{6} - \frac{23}{\sqrt{2} + \sqrt{3} + \sqrt{6}}$ is:

1) 15

2) 16

3) 12

4) 10

$$\begin{aligned} 8.6 + 9.8 - 2.45 - \frac{23}{5.6} &\Rightarrow \sqrt{2} = 1.414 \\ 16 - 4 \approx 12 &\quad \sqrt{3} = 1.732 \\ \therefore \boxed{12} &\quad \sqrt{5} = 2.23 \\ &\quad \sqrt{6} = 2.45 \\ &\quad \sqrt{10} = 3.16 \end{aligned}$$

OR

$$\begin{aligned} 5\sqrt{3} + 7\sqrt{2} - \sqrt{6} - \frac{23(\sqrt{2} + \sqrt{3} - \sqrt{6})}{(\sqrt{2} + \sqrt{3})^2 - (\sqrt{6})^2} \\ \hookrightarrow \frac{23(\sqrt{2} + \sqrt{3} - \sqrt{6})(2\sqrt{6} + 1)}{(2\sqrt{6} - 1) \times (2\sqrt{6} + 1)} \\ \therefore 5\sqrt{3} + 7\sqrt{2} - \sqrt{6} - \frac{23(\sqrt{2} + \sqrt{3} - \sqrt{6})(2\sqrt{6} + 1)}{25} \\ \hookrightarrow 5\sqrt{3} + 7\sqrt{2} - \sqrt{6} - [4\sqrt{3} + \sqrt{2} + 6\sqrt{2} + \sqrt{3} - 12 - \sqrt{6}] \\ \hookrightarrow 5\sqrt{3} + 7\sqrt{2} - \sqrt{6} - 5\sqrt{3} - 7\sqrt{2} + 12 + \sqrt{6} = \boxed{12} \end{aligned}$$

50. The value of $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$ lies between:

1) 2 and 2.5

2) 3 and 3.5

3) 1.5 and 2

4) 2.5 and 3

$$\frac{(6+2\sqrt{5}) - (6-2\sqrt{5})}{4} = \frac{4\sqrt{5}}{4} = \sqrt{5} \approx 2.23$$

$\therefore \text{B/W } 2 \text{ & } 2.5$

51. $\frac{12}{3+\sqrt{5}+2\sqrt{2}} = ?$

1) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$

2) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

3) $1+\sqrt{5}-\sqrt{2}-\sqrt{10}$

4) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

$$\begin{aligned} \sqrt{2} &= 1.4 \\ \sqrt{5} &= 2.2 \\ \sqrt{10} &= 3.2 \end{aligned}$$

$\frac{12}{3+2.2+2.8} = \frac{12}{8} \approx 1.5$

$(b) 1+2.2+1.4-3.2 \approx 4.6-3.2 \approx 1.4 \quad \therefore \boxed{B}$



52. If $(\sqrt{2} + \sqrt{5} - \sqrt{3}) \times k = -12$, then what will be the value of k?

1) $(\sqrt{2} + \sqrt{5} - \sqrt{3})(2 + \sqrt{5})$

2) $(\sqrt{2} + \sqrt{5} + \sqrt{3})(2 - \sqrt{5})$

3) $(\sqrt{2} + \sqrt{5} + \sqrt{3})(2 - \sqrt{10})$

4) $(\sqrt{2} + \sqrt{5} + \sqrt{3})$

$$\sqrt{2} = 1.4$$

$$1.4 \times k = -12 \quad \therefore k = -6$$

$$\sqrt{5} = 2.2$$

(a) x (d) x → +ve Ans नहीं है।

$$\sqrt{3} = 1.7$$

(b) → $5.3 (-0.2) \neq -6$

$$\sqrt{10} = 3.2$$

$$\therefore \text{Ans} = \boxed{C}$$

53. Find the value of $\sqrt{1 + 2019\sqrt{1 + 2020\sqrt{1 + 2021 \times 2023}}}$

1) 2020

2) 2021

3) 2023

4) 2018

$$\therefore \sqrt{1 + 2021 \times 2023} = 2022$$

$$\therefore \sqrt{1 + 2020 \times 2022} = 2021$$

$$\therefore \sqrt{1 + 2019 \times 2021} = \boxed{2020}$$



54. $(\sqrt{6} + \sqrt{10} - \sqrt{21} - \sqrt{35})(\sqrt{6} - \sqrt{10} + \sqrt{21} - \sqrt{35}) = ?$

1) 13

2) 12

3) 11

4) 10

$$\{(\sqrt{6} - \sqrt{35}) + (\sqrt{10} - \sqrt{21})\} \times \{(\sqrt{6} - \sqrt{35}) - (\sqrt{10} - \sqrt{21})\}$$

$$\hookrightarrow (\sqrt{6} - \sqrt{35})^2 - (\sqrt{10} - \sqrt{21})^2$$

$$\hookrightarrow (41 - 2\sqrt{210}) - (31 - 2\sqrt{210}) = \boxed{10}$$

55. $\frac{1}{\sqrt{2} + \sqrt{3} - \sqrt{5}} + \frac{1}{\sqrt{2} - \sqrt{3} - \sqrt{5}} = ?$

1) 1

2) 0

3) $\frac{1}{\sqrt{2}}$

4) $\sqrt{2}$

$$\frac{1}{(\sqrt{2} - \sqrt{5}) + \sqrt{3}} + \frac{1}{(\sqrt{2} - \sqrt{5}) - \sqrt{3}} = \frac{2\sqrt{2} - 2\sqrt{5}}{(\sqrt{2} - \sqrt{5})^2 - \sqrt{3}^2}$$

$$= \frac{2(\sqrt{2} - \sqrt{5})}{7 - 2\sqrt{10} - 3} = \frac{2(\sqrt{2} - \sqrt{5})}{4 - 2\sqrt{10}} = \frac{2(\sqrt{2} - \sqrt{5})}{4(2 - \sqrt{10})}$$

$$= \frac{\sqrt{2} - \sqrt{5}}{\sqrt{2}(2 - \sqrt{5})} = \boxed{\frac{1}{\sqrt{2}}}$$

56. If $a = \sqrt{\frac{15}{16}}$, then $(\sqrt{1+a} + \sqrt{1-a}) = ?$

1) $\sqrt{3}$

2) $\sqrt{\frac{5}{2}}$

3) $\frac{3}{\sqrt{2}}$

4) $\sqrt{2}$

$$a = \frac{\sqrt{15}}{4}$$

$$(\sqrt{1+a} + \sqrt{1-a})^2 = 1+a+1-a+2\sqrt{1-a^2}$$

$$a^2 = \frac{15}{16}$$

$$\hookrightarrow 2 + 2\sqrt{1-\frac{15}{16}} \rightarrow 2 + 2 \times \frac{1}{4} \rightarrow \frac{5}{2}$$

$$\therefore (\sqrt{1+a} + \sqrt{1-a})^2 = \frac{5}{2}$$

$$\therefore (\sqrt{1+a} + \sqrt{1-a}) = \boxed{\sqrt{\frac{5}{2}}}$$

57. If $x = \sqrt{1+\frac{\sqrt{3}}{2}} - \sqrt{1-\frac{\sqrt{3}}{2}}$, then the value of $\frac{\sqrt{2}-x}{\sqrt{2}+x}$ will be closest to:

1) 0.12

2) 1.4

3) 0.17

4) 1.2

$$x^2 = 2 - 2\sqrt{1-\frac{3}{4}} = 2 - 2 \times \frac{1}{2} = 1 \rightarrow x = 1$$

$$\therefore \frac{\sqrt{2}-1}{\sqrt{2}+1} \rightarrow \frac{(\sqrt{2}-1)^2}{1} \rightarrow 3-2\sqrt{2} \rightarrow 3-2\cdot 82 \approx 0.18$$

$$\therefore \boxed{0.17}$$

58. $\frac{\sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}} = ?$

1) 1

2) -1

3) 2

4) -2

$$\sqrt{3-2\sqrt{2}} = \sqrt{(\sqrt{2}-1)^2} = (\sqrt{2}-1)$$

$$x = \frac{\sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$$

$$x^2 = \frac{2\sqrt{5} + 2\sqrt{5-4}}{\sqrt{5}+1} \rightarrow \frac{2(\sqrt{5}+1)}{(\sqrt{5}+1)} = 2$$

$$\therefore x = \sqrt{2}$$

$$\therefore \sqrt{2} - (\sqrt{2}-1) = \boxed{1}$$

59. What is the value $\sqrt[3]{(26+15\sqrt{3})} + \sqrt[3]{(26-15\sqrt{3})} = ?$

1) 6

2) 5

3) 4

4) 3

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$x = \sqrt[3]{(26+15\sqrt{3})} + \sqrt[3]{(26-15\sqrt{3})}$$



$$x^3 = 26 + 15\sqrt{3} + 26 - 15\sqrt{3} + 3 \times \sqrt[3]{676 - 675} \quad (\text{x})$$

$$x^3 = 52 + 3 \times 1 \quad (\text{x})$$

$$x^3 = 52 + 3x \quad \therefore \text{from options } \boxed{x=4}$$

60. If $\sqrt{5x-6} + \sqrt{5x+6} = 6$, then $x = ?$

- 1) -4 2) 0 3) 2 4) 4

from options $\rightarrow x=2$

$$\sqrt{10-6} + \sqrt{10+6} = 2+4 = 6 \quad \therefore \boxed{x=2}$$

61. If $\sqrt{6x-17} + \sqrt{6x+17} = 5(\sqrt{2}+1)$, then the value of $x = ?$

- 1) 7 2) 8 3) 6 4) 5

$$(a) \rightarrow \sqrt{42-17} + \sqrt{50} = 5 + 5\sqrt{2} = 5(\sqrt{2}+1) \quad \checkmark \quad \therefore \boxed{x=7}$$

62. If $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} + \frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = 194$, then $x = ?$

- 1) 7/2 2) 4 3) 7 4) 14

$$\frac{(x+\sqrt{x^2-1})^2 + (x-\sqrt{x^2-1})^2}{x^2 - (x^2-1)} = 194 \quad (a+b)^2 + (a-b)^2 = 2(a^2+b^2)$$

$$\therefore x^2 - (x^2-1) = 1$$

$$\therefore 2(x^2 + x^2 - 1) = 194 \quad \rightarrow \quad 4x^2 = 194 \quad \therefore \boxed{x=7}$$

63. If $\frac{\sqrt{5+x} + \sqrt{5-x}}{\sqrt{5+x} - \sqrt{5-x}} = 3$, then what is the value of x ?

- 1) 5/2 2) 25/3 3) 4 4) 3

from options $\rightarrow x=3$

$$\frac{2\sqrt{2} + \sqrt{2}}{2\sqrt{2} - \sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3 \quad \therefore \boxed{x=3}$$

64. If $p = \frac{\sqrt{3q+2} + \sqrt{3q-2}}{\sqrt{3q+2} - \sqrt{3q-2}}$ then the value of $P^2 - 3pq + 2$ is equal to:

- 1) 0 2) 1 3) 2 4) 3

$$\frac{P}{1} = \frac{\sqrt{3q+2} + \sqrt{3q-2}}{\sqrt{3q+2} - \sqrt{3q-2}}$$

Apply componendo & dividendo

$$\frac{P+1}{P-1} = \frac{\sqrt{3q+2}}{\sqrt{3q-2}} \quad \rightarrow \text{Squaring} \quad \frac{(P+1)^2}{(P-1)^2} = \frac{3q+2}{3q-2}$$



Apply again

$$\frac{(p+1)^2 + (p-1)^2}{(p+1)^2 - (p-1)^2} = \frac{3q}{2} \rightarrow \frac{2(p^2 + 1)}{4p} = \frac{3q}{2}$$

$$\hookrightarrow p^2 + 1 = 3pq \Rightarrow p^2 - 3pq + 2 = 1$$

$$p^2 - 3pq + 1 = 0$$

65. What is the value of x, if $\frac{b + \sqrt{b^2 - 2bx}}{b - \sqrt{b^2 - 2bx}} = a$?

1) $\frac{ab}{a+b}$

2) $\frac{2ab}{a+1}$

3) $\frac{2ab}{(a+1)^2}$

4) $\frac{ab}{(a+1)^2}$

Apply componendo & dividendo

$$\frac{xb}{\cancel{x}\sqrt{b^2 - 2bx}} = \frac{a+1}{a-1} \rightarrow \text{squaring} \quad \frac{b^2}{b^2 - 2bx} = \frac{(a+1)^2}{(a-1)^2}$$

Apply again

$$\frac{2b^2 - 2bx}{2bx} = \frac{2(a^2 + 1)}{4a} \rightarrow \frac{b^2 - bx}{bx} = \frac{a^2 + 1}{2a}$$

$$\frac{\cancel{b}(b-x)}{bx} = \frac{a^2 + 1}{2a} \rightarrow \frac{b}{x} - 1 = \frac{a^2 + 1}{2a}$$

$$\frac{b}{x} = \frac{a^2 + 1}{2a} + 1 \rightarrow \frac{b}{x} = \frac{(a+1)^2}{2a} \therefore x = \boxed{\frac{2ab}{(a+1)^2}}$$



66. If $5^{x-1} - 5^{x-1} = 600$, then what is the value of 10^{2x} ?

1) 1

2) 1000

3) 100000

4) 1000000

$10^{2x} \rightarrow$ even no. of zero at the end

$$5^{x+1} - 5^{x-1} = 600$$

$$x=3 ; 5^4 - 5^2 = 625 - 25 = 600 \therefore x=3$$

$$\therefore 10^6 = \boxed{1000000}$$

OR $5^x \times 5^1 - \frac{5^x}{5^1} = 5^x (5 - \frac{1}{5}) = 600$

$$5^x (\frac{24}{5}) = 600 \rightarrow 5^x = \frac{600 \times 5}{24-2} = 125$$

$$\therefore 5^x = 5^3 \therefore \boxed{x=3}$$



67. If $24^x + 27^{[x-\left(\frac{1}{3}\right)]} = 972$, then what is the value of x?

- 1) 2 2) 3 3) 4 4) 5

$$27^x + 27^{[x-\frac{1}{3}]} = 972 \quad \therefore \text{ Only } [x=2] \text{ possible}$$

$$27^2 + 3^{3x\frac{5}{3}} = 27^2 + 3^5 = 729 + 243 = 972$$

68. If $25^{x-1} = 5^{2x-1} - 100$, then what is the value of x?

- 1) 1 2) 2 3) -2 4) -3

from options $\rightarrow x=2 \quad 25 = 125-100 \quad (\checkmark) \quad \therefore [x=2]$

69. If $x(2x+3) = 90$ and $7y^{\frac{-1}{2}} + 2y^{\frac{-1}{2}} = y^{\frac{1}{2}}$ (x and y are positive numbers), then what is the value of $(x^2 + y^2) = ?$

- 1) 45 2) 109 3) 117 4) 126

$$x(2x+3) = 90$$

$$\boxed{x=6}$$

$$\frac{g}{\sqrt{y}} = \sqrt{y} \quad \therefore \boxed{y=9}$$

$$x^2+y^2 = 36+81 = \boxed{117}$$



70. If $9^{2x-1} - 81^{x-1} = 1944$, then x is

- 1) 3 2) $\frac{9}{4}$ 3) $\frac{4}{9}$ 4) $\frac{1}{3}$

from options (a) $9^5 - 81^2 = 9^5 - 9^4$
 $9^5 - 9^4 = 9^4(9-1) = 8 \quad (\times)$

(b) $9^{\frac{1}{2}} - 81^{\frac{5}{4}} \rightarrow \frac{2x\frac{1}{2}}{3} - \frac{4x\frac{5}{4}}{3} \rightarrow 3^7 - 3^5$
 $3^7 - 3^5 = 8-1 \quad (\checkmark) \quad \therefore \boxed{x = \frac{9}{4}}$

71. $9^{\frac{x-1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$ then x is

- 1) $\frac{3}{2}$ 2) $\frac{2}{5}$ 3) $\frac{3}{4}$ 4) $\frac{4}{9}$

(b) $\times \quad (d) \times \quad \frac{3}{5}, \frac{4}{9} \quad (\times) \quad \text{क्योंकि नीचे बड़ा no. है} \quad |$

(a) $\rightarrow 9^1 - 2^1 = 2^{\frac{x \times 3}{2}} - 3^0 \rightarrow 7 = 8-1 \quad (\checkmark)$
 $\therefore \boxed{x = \frac{3}{2}}$

72. If $5^x - 3^y = 13438$ and $5^{x-1} + 3^{y+1} = 9686$, then x + y equals

- 1) 11 2) 12 3) 13 4) 14

$$\begin{aligned}
 3 \times 5^x - 3^{y+1} &= 13438 \times 3 = 40314 \\
 5^{x-1} + 3^{y+1} &= 9686 \\
 \hline
 3 \times 5^x + 5^{x-1} &= 50000 \\
 3 \times 5^x + \frac{5^x}{5} &\rightarrow 5^x \left(3 + \frac{1}{5}\right) \rightarrow 5^x \times \frac{16}{5} = \underline{\underline{50,000}}
 \end{aligned}$$

$$\hookrightarrow 5^x = 3125 \times 5 \rightarrow 5^x = 5^6 \therefore \boxed{x=6}$$

$$\therefore 5^6 - 3^y = 13438 \rightarrow 15625 - 3^y = 13438$$

$$\begin{aligned}
 \hookrightarrow 2187 = 3^y \rightarrow 3^7 = 3^y \therefore \boxed{y=7} && 9^3 = 729 \\
 x+y = 6+7 = \boxed{13} && 3^6 = \frac{729}{x^3} \\
 && 3^7 = \cancel{x^3} \rightarrow \cancel{2187}
 \end{aligned}$$

OR

$$\begin{aligned}
 5^x - 3^y &= 13438 \quad \therefore 5^x > 13438 \quad 3^3 = 27 \\
 \downarrow &\quad \downarrow & 5^6 = 15625 & 3^7 = 2187 \\
 \dots 5 - \dots 7 &= \textcircled{8} & & \\
 x=6, y=7 &\quad \therefore \boxed{x+y=13} & &
 \end{aligned}$$

73. If $9^x 3^y = 2187$ and $2^{3x} 2^{2y} - 4^{xy} = 0$, then $x + y = ?$

1) 4

2) 3

3) 5

4) 7

$$\begin{aligned}
 3^{2x+y} &= 3^7 \\
 \therefore 2x+y &= 7 \\
 2x\textcircled{2} + \textcircled{3} &= 7 \\
 \therefore x=2, y=3 &\quad \therefore \boxed{x+y=5}
 \end{aligned}$$

$$\begin{aligned}
 2^{3x+2y} &= 2^{2xy} \\
 \therefore 3x+2y &= 2xy \\
 3x2 + 2x3 &= 2 \times 2 \times 3 \\
 6+6 &= 12
 \end{aligned}$$



74. Let $0 < x < 1$. Then the correct inequality is

- 1) $x < \sqrt{x} < x^2$ 2) $\sqrt{x} < x < x^2$ 3) $x^2 < x < \sqrt{x}$ 4) $\sqrt{x} < x^2 < x$

$$x = 0.25 \text{ (let)}$$

$$\sqrt{x} = 0.5 \text{ (↑)}$$

$$x^2 = 0.0625 \text{ (↓)}$$

$$\therefore \boxed{x^2 < x < \sqrt{x}}$$

for nos b/w 0 and 1

$N^2 \rightarrow$ decrease

$\sqrt{N} \rightarrow$ increase.

75. If $3^x = 4^y = 12^z$, then z is equal to?

1) xy

2) $x+y$

3) $\frac{xy}{x+y}$

4) $4x + 3y$

$$3^x = 4^y = 12^z = K \text{ (let)}$$

$$3^x = K \quad \therefore 3 = K^{\frac{1}{x}}$$

$$4^y = K \quad \therefore 4 = K^{\frac{1}{y}} \quad] \text{ multiply}$$

$$12^z = K \quad \therefore 12 = K^{\frac{1}{z}}$$

$$3 \times 4 = K^{\frac{1}{x}} \times K^{\frac{1}{y}}$$

$$12 = K^{\frac{1}{x} + \frac{1}{y}}$$

$$K^{\frac{1}{z}} = K^{\frac{1}{x} + \frac{1}{y}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \rightarrow \frac{y+x}{xy} \rightarrow \frac{1}{z} \quad \therefore z = \frac{xy}{x+y}$$

OR

$$3^1 \times 4^1 = 12^1 \quad * \quad 2^x = 3^y = 12^z$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

$$2^{\textcircled{1}} \times 3^{\textcircled{2}} = 12^{\textcircled{1}}$$

$$\therefore \frac{3}{x} + \frac{2}{y} = \frac{1}{z}$$

76. If $2^x = 3^y = 18^z$, then which one is correct ?

$$1) \frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 0$$

$$2) \frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$3) \frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 0$$

$$4) \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 0$$

$$2^1 \times 3^2 = 18^1$$

$$\frac{1}{x} + \frac{2}{y} = \frac{1}{z} \quad \therefore \quad \frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 0$$

77. If $(5.55)^x = (0.555)^y = 1000$, then the value of $\frac{1}{x} - \frac{1}{y}$ is

1) 3

2) 1

3) 1/3

4) 2/3

$$\frac{5.55}{0.555} = \frac{1000^{\frac{1}{x}}}{1000^{\frac{1}{y}}} \quad \therefore 1000^{\frac{1}{x}} = 1000^{\frac{1}{x}} - \frac{1}{y}$$

$$10 = 1000^{\frac{1}{x}} - \frac{1}{y}$$

$$\therefore \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$$

$$\text{OR} \quad \frac{5.55}{0.555} = 10 \quad \therefore \left(\frac{5.55}{0.555} \right)^3 = 1000 \rightarrow 5.55^3 \times 0.555^{-3} = 1000^1 \quad \therefore \frac{3}{x} + \frac{-3}{y} = 1 \rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$$

78. If $2^x - 2^y = \frac{31}{6}$ and $4^x - 4^y = \frac{341}{12}$, then $(x - y) = ?$

1) 3

2) 4

3) 5

4) 6

$$2^{2x} - 2^{2y} = \frac{341}{12}$$

$$\therefore 2^x + 2^y = \frac{11}{2}$$

$$\hookrightarrow (2^x)^2 - (2^y)^2 = \frac{341}{12}$$

$$2^x - 2^y = \frac{31}{6}$$

~

~



$$\begin{aligned}
 &\rightarrow (2^x + 2^y)(2^x - 2^y) = \frac{341}{12} \\
 &\rightarrow (2^x + 2^y) \times \frac{31}{6} = \frac{341}{12} \\
 &\rightarrow (2^x + 2^y) = \frac{11}{2} \\
 &2^x = \frac{\frac{11}{2} + \frac{31}{6}}{2} = \frac{\frac{32}{6}}{\frac{6}{3}} \times \frac{1}{2} = \frac{16}{3} \\
 &2^y = \frac{\frac{11}{2} - \frac{31}{6}}{2} = \frac{\frac{-2}{2}}{\frac{12}{6}} = \frac{1}{6} \\
 \therefore \frac{2^x}{2^y} = \frac{\frac{16}{3} \times 6}{1} = 32 \quad \therefore 2^{x-y} = 2^5 \quad \therefore \boxed{x-y=5}
 \end{aligned}$$

79. If $a^x = (x+y+z)^y$, $a^y = (x+y+z)^z$, $a^z = (x+y+z)^x$, then which of the following is correct?

- 1) $3(x+y+z) = a$ 2) $x+y+z = 2a$

- 3) $x=y=z = \frac{a}{3}$ 4) $x=y \neq z$

multiply all equations

$$a^{x+y+z} = (x+y+z)^{x+y+z} \rightarrow a = \frac{x+y+z}{\frac{a}{3}, \frac{a}{3}, \frac{a}{3}}$$

$$\therefore x=y=z \rightarrow \boxed{x=y=z = \frac{a}{3}}$$

80. If $x^{y+z} = 1$, $y^{x+z} = 1024$ and $z^{x+y} = 729$ (x , y and z are natural numbers), then what is the value of $(z+1)^{y+x+1}$?

- 1) 6561 2) 10000 3) 4096 4) 14641

$$\begin{aligned}
 x^{y+z} = 1 &\rightarrow x=1 \\
 y^{x+z} = 1024 &\rightarrow y^{1+z} = 2^{10} \rightarrow y=2, z=9 \\
 z^{x+y} = 729 &\rightarrow z^{1+y} = 9^3 \rightarrow z^{1+2}, y=2 \\
 \therefore x=1 & \quad (10)^4 \rightarrow \boxed{10000}
 \end{aligned}$$



81. $\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} = ?$

- 1) $5-3\sqrt{3}$ 2) $5+3\sqrt{3}$ 3) $7-3\sqrt{3}$ 4) $2+3\sqrt{3}$

$$\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} \times \frac{33+19\sqrt{3}}{33+19\sqrt{3}} \Rightarrow \sqrt{\frac{(6+2\sqrt{3})(33+19\sqrt{3})}{6}} \quad \begin{array}{l} 33^2=1089 \\ (19\sqrt{3})^2=1083 \\ \hline 6 \end{array}$$

$$\rightarrow \sqrt{\frac{(3+\sqrt{3})(33+19\sqrt{3})}{3}} \rightarrow \sqrt{\frac{156+90\sqrt{3}}{3}} \rightarrow \sqrt{52+30\sqrt{3}}$$

$$= \boxed{5+3\sqrt{3}}$$

82. $a = \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}}$ and $b = \frac{\sqrt{7} - \sqrt{6}}{\sqrt{7} + \sqrt{6}}$ then find $\frac{a^2}{b} + \frac{b^2}{a} = ?$

- 1) 17498 2) 17550 3) 17654 4) 17576

$$\frac{1}{a} = b \quad \therefore \quad a + \frac{1}{a} = \frac{2(7+6)}{(7-6)} = 26$$

$$\frac{a^2}{b} + \frac{b^2}{a} \Rightarrow a^3 + \frac{1}{a^3} = 26^3 - 3 \times 26 = \boxed{17498}$$

83. What is $\sqrt{1 + \frac{1}{2^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{2007^2} + \frac{1}{2008^2}}$ equal to ?

- 1) $2008 - \frac{1}{2008}$ 2) $2007 - \frac{1}{2007}$ 3) $2007 - \frac{1}{2008}$ 4) $2008 - \frac{1}{2009}$

$$\text{total} = 2007 \text{ term}$$

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{4}} = \frac{3}{2} \quad \therefore (1 + \frac{1}{2}) + (1 + \frac{1}{6}) + (1 + \frac{1}{12}) + \dots \text{..... 2007 times}$$

$$\sqrt{1 + \frac{1}{4} + \frac{1}{9}} = \sqrt{\frac{49}{36}} = \frac{7}{6} \quad \hookrightarrow 2007 + \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + 2007 \text{ terms} \right)$$

$$\sqrt{1 + \frac{1}{9} + \frac{1}{16}} = \sqrt{\frac{169}{144}} = \frac{13}{12} \quad \hookrightarrow 2007 + \left(\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{2007 \times 2008} \right)$$

$$\hookrightarrow 2007 + \left(1 - \frac{1}{2008} \right) = \boxed{2008 - \frac{1}{2008}}$$

84. The value of $\frac{\sqrt{0.6912} + \sqrt{0.5292}}{\sqrt{0.6912} - \sqrt{0.5292}}$ is:

- 1) 1.5 2) 0.9 3) 15 4) 9

$$80^2 = 6400 \quad \therefore 6912 \approx 83^2 \quad 11y \quad 5292 \approx 73^2$$

$$85^2 = 7225 \quad \therefore \frac{83+73}{83-73} = \frac{156}{10} \approx 15.6 = \boxed{15}$$

OR Simplify by 12 $\rightarrow \frac{\sqrt{516} + \sqrt{441}}{\sqrt{516} - \sqrt{441}} = \frac{24+21}{24-21} = \frac{45}{3} = \boxed{15}$

85. $\sqrt{12\sqrt{12\sqrt{12\sqrt{12\dots\alpha}}}} = ?$

- 1) 8 2) 12 3) 36 4) 6



$$x = \sqrt{12} \sqrt{12} \sqrt{12} \dots \infty \quad \text{OR} \quad \infty \text{ multiply series with only one number}$$

$$x^2 = 12x$$

$$\boxed{x=12}$$

Ans \rightarrow same no.
 $\therefore \boxed{12}$

86. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\dots\alpha}}}} = 343^{y-1}$ then $y = ?$

- 1) 4/3 2) 3/2 3) 5/4 4) 1

$$\therefore 7^1 = 7^{3(y-1)} \rightarrow 3(y-1) = 1 \rightarrow 3y-3 = 1 \quad \therefore \boxed{y=\frac{4}{3}}$$

87. $\sqrt[3]{64\sqrt[3]{64\sqrt[3]{64\dots\infty}}}$?

- 1) 4 2) 8 3) 16 4) $4\sqrt{2}$

$$x = \sqrt[3]{64} \sqrt[3]{64\dots} \rightarrow x = \sqrt[3]{64x}$$

$$x^3 = 64x$$

$$x^2 = 64$$

$$x = \boxed{8}$$

88. Solve $\sqrt[5]{16\sqrt[5]{16\sqrt[5]{16\dots\infty}}}$?

- 1) 2 2) 3 3) 4 4) $16^{\frac{1}{6}}$

$$\rightarrow (5-1)\sqrt[5]{16} \Rightarrow \sqrt[4]{16} \Rightarrow (2^4)^{\frac{1}{4}} \Rightarrow \boxed{2}$$

89. If $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}}$, then what is the value of m?

- 1) $\frac{1}{8}$ 2) $\frac{1}{4}$ 3) $\frac{3}{4}$ 4) $\frac{7}{4}$

$$\begin{aligned} & \sqrt[14]{x\sqrt{x\sqrt{x}}} \rightarrow x^{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{14}} = x^{\frac{1}{56}} \\ & \rightarrow x^{\frac{1}{2} \times \frac{1}{4}} = x^{\frac{1}{28}} \end{aligned}$$

$$x^m = x^{\frac{1}{56} + \frac{1}{28} + \frac{1}{14}} = x^{\frac{1+2+4}{56}} = x^{\frac{1}{8}}$$

$$\therefore x^m = x^{\frac{1}{8}} \quad \therefore \boxed{m = \frac{1}{8}}$$



90. $\sqrt{27 \div \sqrt{27 \div \sqrt{27 \div \sqrt{27 \dots \dots \infty}}}} = ?$

1) 3

2) $3\sqrt{3}$

3) $\sqrt{3}$

4) 9

$$x = \sqrt{27 \div \sqrt{27 \div \sqrt{27 \dots \dots \infty}}} \\ x = \sqrt{\frac{27}{x}} \rightarrow x^2 = \frac{27}{x} \rightarrow x^3 = 27 \therefore x = 3$$

91. $x^{\sqrt{x^{\sqrt{x^{\dots \dots \infty}}}}} = \frac{1}{2}$ then $x = ?$

1) $\frac{1}{8}$

2) $\frac{1}{4}$

3) $\frac{1}{16}$

4) $\frac{1}{32}$

$$\sqrt{x^{\frac{1}{2}}} = \frac{1}{2} \rightarrow x^{\frac{1}{4}} = \frac{1}{2} \rightarrow x = \left(\frac{1}{2}\right)^4 = \boxed{\frac{1}{16}}$$

92. If $x^{\sqrt{x^{\sqrt{x^{\dots \dots \infty}}}}} = \frac{1}{36}$ then $x = ?$

1) 6^{-12}

2) 6^{-18}

3) 6^{-8}

4) 6^{-16}

$$(\sqrt{x} \times \sqrt{x})^{\sqrt{x}^{\dots \dots \infty}} = \frac{1}{36} \rightarrow \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\dots \dots}}} \times \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\dots \dots}}} = \frac{1}{6} \times \frac{1}{6} \\ \therefore \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\dots \dots}}} = \frac{1}{6} \rightarrow \sqrt{x}^{\frac{1}{6}} = \frac{1}{6} \rightarrow x^{\frac{1}{12}} = \frac{1}{6} \\ \therefore x = \left(\frac{1}{6}\right)^{12} = \boxed{6^{-12}}$$

93. $\sqrt{12 \sqrt{12 \sqrt{12 \sqrt{12 \sqrt{12 \sqrt{12}}}}} = ?$

1) $12^{\frac{32}{31}}$

2) $12^{\frac{64}{63}}$

3) $12^{\frac{31}{32}}$

4) $12^{\frac{63}{64}}$

$$12^{\frac{\frac{6}{6}-1}{2^6}} = \boxed{12^{\frac{63}{64}}}$$

94. $\sqrt[3]{11 \sqrt[3]{11 \sqrt[3]{11 \sqrt[3]{11 \sqrt[3]{11}}}}} = 121^k$ then $k = ?$

1) $\frac{121}{486}$

2) $\frac{129}{489}$

3) $\frac{123}{509}$

4) $\frac{119}{477}$

$$11^{\frac{1}{243} + \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \frac{1}{3}}$$

$$\hookrightarrow 11^{\frac{121}{243}} = 11^{2k} \therefore 2k = \frac{121}{243} \rightarrow k = \boxed{\frac{121}{486}}$$



95. $\sqrt[m]{a^n} \sqrt[m]{b} \sqrt[m]{a^n} \sqrt[m]{b} \sqrt[m]{a^n} b \dots \infty$ can be written as

1) $\sqrt[mn-1]{a^n b}$

2) $\sqrt[mn]{ab}$

3) $\sqrt[mn-1]{b^n a}$

4) $\sqrt[mn+1]{a^n b}$

$$x = \sqrt[m]{a} \sqrt[n]{b} x$$

$$\therefore x^{mn-1} = a^n b$$

$$x^m = \sqrt[m]{b} x$$

$$x^{mn} = a^n x b x$$

$$x = \boxed{\sqrt[mn-1]{a^n b}}$$

96. Find $\sqrt{2 \times \sqrt[3]{4 \times \sqrt[3]{2 \times \sqrt[3]{4 \times \sqrt[3]{4 \dots \infty}}}} = ?$

1) $\sqrt{2}$

2) 2

3) 4

4) $4\sqrt{2}$

$$\sqrt[2 \times 3 - 1]{2^3 \times 4} \Rightarrow \sqrt[5]{32} \Rightarrow \boxed{2}$$

$$\Rightarrow \sqrt{a + \sqrt{a + \sqrt{a + \dots}}} = \frac{\sqrt{4a+1} + 1}{2} = x$$

$$\sqrt{a - \sqrt{a - \sqrt{a - \dots}}} = \frac{\sqrt{4a+1} - 1}{2} = y$$

$$\begin{aligned} x-y &= 1 \\ xy &= a \end{aligned}$$

* $x+y = \sqrt{4a+1}$

OR Take two factors of a whose diff is 1 like $56 \leftarrow \frac{8}{7}$ $72 \leftarrow \frac{8}{9}$

Then $x = \text{larger factor}$
 $y = \text{smaller factor}$

97. Find the value of $\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$

1) 5

2) $3\sqrt{10}$

3) 6

4) 7

$$30 \leftarrow \frac{6}{5} \therefore \boxed{6}$$

98. Let $x = \sqrt{272 + \sqrt{272 + \sqrt{272 + \sqrt{272 + \dots}}}}$ to infinity;

1) 16

2) $4\sqrt{13}$

3) 17

4) 4.35

$$272 \leftarrow \frac{16}{17} \therefore \boxed{17}$$

99. What is the value of $2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} = ?$

1) 1

2) 2

3) 3

4) 4

$$2 + 2 = \boxed{4}$$



100. Let $x = \sqrt{42 - \sqrt{42 - \sqrt{42 - \sqrt{42 - \dots}}}}$ to infinity then x equals to..

1) 6

2) 7

3) Between 6 and 7

4) Greater than 7

$$42 < \frac{6}{7} \quad \therefore \boxed{6}$$

101. $\frac{\sqrt{210 + \sqrt{219 + \sqrt{210 + \dots}}}}{\sqrt{156 - \sqrt{156 - \sqrt{156 - \dots}}}} = ?$

1) 1

2) 1.33

3) 1.25

4) 1.5

$$\frac{210}{15 \times 14} < \frac{156}{13 \times 12} \quad \therefore \quad \frac{15}{12} = \frac{5}{4} = \boxed{1.25}$$

102. $\frac{\sqrt{8\sqrt{8\sqrt{8\sqrt{8}}}} * \sqrt{56 + \sqrt{56 + \sqrt{56 + \dots}}}}{\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} * \sqrt{20 - \sqrt{20 - \sqrt{20 - \dots}}}} = 2^b$ then b = ?

1) $\frac{95}{32}$

2) $\frac{93}{32}$

3) $\frac{99}{32}$

4) $\frac{91}{32}$

$$\begin{aligned} \frac{8^{\frac{31}{32}} \times 8^{\frac{1}{2}}}{2^{\frac{15}{16}} \times \cancel{4}} &\Rightarrow 8^{\frac{31}{32}} \times 2^{1 - \frac{15}{16}} \Rightarrow 2^{\frac{3 \times 31}{32}} \times 2^{\frac{1}{16}} \\ \Rightarrow 2^{\frac{93}{32} + \frac{1}{16}} &\Rightarrow 2^{\frac{95}{32}} \quad \therefore \boxed{b = \frac{95}{32}} \end{aligned}$$

103. Find $\sqrt{31 + \sqrt{31 + \sqrt{31 + \sqrt{31 + \dots}}}} = ?$

1) $5\sqrt{5} - 1.5$

2) $2.5\sqrt{5} + 0.5$

3) $\frac{5\sqrt{5} - 1}{2}$

4) $\frac{2\sqrt{31} + 1}{2}$

$$\frac{\sqrt{4 \times 31 + 1} + 1}{2} = \frac{5\sqrt{5} + 1}{2} = \boxed{2.5\sqrt{5} + 0.5}$$

104. Find $\sqrt{14 + \sqrt{14 + \sqrt{14 + \sqrt{14 + \dots}}}} = ?$

1) 4 and 4.5

2) 4.5 and 5

3) 3 and 4

4) None

$$\frac{\sqrt{4 \times 14 + 1} + 1}{2} = \frac{\sqrt{57} + 1}{2}$$

$\sqrt{57}$
smaller larger
 $\sqrt{49}$ $\sqrt{64}$

$$\therefore \frac{7+1}{2} \neq \frac{8+1}{2} \Rightarrow \boxed{4 \text{ & } 4.5}$$



105. Find $\sqrt{19 - \sqrt{19 - \sqrt{19 - \sqrt{19 - \dots}}}} = ?$

1) $\frac{\sqrt{77} - 1}{2}$

2) $\frac{\sqrt{19} + 3}{2}$

3) $\frac{\sqrt{77} + 1}{2}$

4) Between 4 and 5

$$\frac{\sqrt{4 \times 19 + 1} - 1}{2} = \frac{\sqrt{77} - 1}{2}$$

106. If $A = \sqrt{10 - \sqrt{10 - \sqrt{10 - \sqrt{10 - \dots}}}}$ then which of the following is true?

1) $A = 2.5$

2) $2.5 < A < 3$

3) $\frac{\sqrt{41} - 3}{2}$

4) Greater than 3

$$A = \frac{\sqrt{41} - 1}{2} \therefore \frac{6-1}{2} = 2.5 \text{ & } \frac{7-1}{2} = 3$$

$\therefore 2.5 < A < 3$

107. $a = \sqrt{13 + \sqrt{13 + \sqrt{13 + \sqrt{13 + \dots}}}}$ and $b = \sqrt{13 - \sqrt{13 - \sqrt{13 - \sqrt{13 - \dots}}}}$, then which option is true?

1) $a+b+1=0$

(b) $a-b-1=0$

(c) $a-b+1=0$

(d) $a-b-1=0$

$$a-b=1 \quad \therefore \boxed{a-b-1=0}$$

108. The value of $p + \sqrt{p^2 + \sqrt{p^4 + \sqrt{p^3 + \sqrt{p^8 + \sqrt{p^{16} + \dots}}}}}$

1) $p\left(\frac{\sqrt{5}+2}{2}\right)$

2) $p\left(\frac{3+\sqrt{5}}{2}\right)$

3) $p\left(\frac{p}{1+\sqrt{p}}\right)$

4) $p\left(\frac{\sqrt{5}+1}{2}\right)$

Take $p=1 \quad \therefore 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$

$\hookrightarrow 1 + \frac{\sqrt{5}+1}{2} = \frac{\sqrt{5}+3}{2} \quad \therefore \text{option(b)} \checkmark$



109. Find $\sqrt{35 + 2\sqrt{35 + \sqrt{2\sqrt{35 + 2\sqrt{35 + \dots}}}}} = ?$

1) 6

2) 7

3) 5

4) 6.4

$x = \sqrt{35 + 2x}$

OR

$$\frac{\sqrt{4 \times 35 + 4} + 2}{2}$$

$x^2 = 35 + 2x$

$$\frac{\sqrt{144} + 2}{2} = \frac{14}{2} = \boxed{7}$$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7, -5$

$\therefore \boxed{x=7}$

110. Find $\sqrt{154 + 3\sqrt{154 + 3\sqrt{154 + 3\sqrt{154 + \dots}}}}$

1) 13

2) 14

3) 11

4) $\frac{\sqrt{613} + 9}{2}$

factor (diff = 3) $\therefore 154 \leftarrow 14 \times 11 \quad \therefore \boxed{14}$

111. Find $\sqrt{154 - 3\sqrt{154 - 3\sqrt{154 - 3\sqrt{154 - \dots}}}}$

1) 13

2) 14

3) 11

4) $\frac{\sqrt{613} + 9}{2}$

$154 \leftarrow 14 \times 11 \quad \therefore \boxed{11}$

112. Find $\sqrt{3 + 4\sqrt{3 + 4\sqrt{3 + 4\sqrt{3 + \dots}}}} = ?$

1) $\sqrt{7} + 2$

2) $2\sqrt{7} - 3$

3) $2\sqrt{7}$

4) $4 + \sqrt{7}$

$$\frac{\sqrt{4 \times 3 + 4^2} + 4}{2} \Rightarrow \frac{2\sqrt{7} + 4}{2} \Rightarrow \boxed{\sqrt{7} + 2}$$

113. Find $\sqrt{5 - 2\sqrt{5 - 2\sqrt{5 - 2\sqrt{5 - \dots}}}}$

1) $\sqrt{2} + 1$

2) $\sqrt{6} - 1$

3) $2\sqrt{2} - 5$

4) $\sqrt{5} - 2$

$$\frac{\sqrt{4 \times 5 + 2^2} - 2}{2} \Rightarrow \frac{2\sqrt{6} - 2}{2} \Rightarrow \boxed{\sqrt{6} - 1}$$

* $x-y=2$
 $xy = 5$

$$x+y = \sqrt{(x-y)^2 + 4xy} = 2\sqrt{6}$$

$$\therefore x = \sqrt{6} + 1 \\ y = \sqrt{6} - 1$$



114. $P = \sqrt{11 + 3\sqrt{11 + 3\sqrt{11 + 3\sqrt{11 + 3\sqrt{11 + \dots}}}}}$ and $Q = \sqrt{11 - 3\sqrt{11 - 3\sqrt{11 - 3\sqrt{11 - 3\sqrt{11 - \dots}}}}}$

then $P + Q = ?$

1) $\sqrt{47}$

2) $\sqrt{65}$

3) $\sqrt{41}$

4) $\sqrt{53}$

$P-Q = 3$

$PQ = 11$

$$P+Q = \sqrt{(P-Q)^2 + 4PQ} = \boxed{\sqrt{53}}$$

OR $P+Q = \sqrt{4 \times 11 + 3^2} = \sqrt{53}$



115. Let $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \dots}}}}$ then x equals

(a) 3

(b) $\sqrt{13}$

(c) $\frac{\sqrt{13}-1}{2}$

(d) $\frac{\sqrt{13}+1}{2}$

$$\frac{\sqrt{4x4-3} + 1}{2} = \boxed{\frac{\sqrt{13} + 1}{2}}$$

116. Let $x = \sqrt{6 - \sqrt{6 + \sqrt{6 - \sqrt{6 + \dots}}}}$ then x equals

1) 3

2) $\sqrt{21}$

3) $\frac{\sqrt{21}-1}{2}$

4) $\frac{\sqrt{21}+1}{2}$

$$\frac{\sqrt{4x6-3} - 1}{2} = \boxed{\frac{\sqrt{21} - 1}{2}}$$

117. If $p = \sqrt{12 + 13\sqrt{12 + 13\sqrt{12 + \dots}}}$ and $q = \sqrt{12 - 13\sqrt{12 - 13\sqrt{12 - \dots}}}$ then find $p - q + pq$?

1) 24

2) 25

3) 26

4) 27

$$P-Q = 13 \quad \therefore P-Q + PQ = 13+12 = \boxed{25}$$

$$PQ = 12$$

118. If $x = \sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{x + \dots}}}}$ then find x ?

1) 1

2) 2

3) 3

4) 4



$$x = \frac{\sqrt{4x4+4} + 2}{2}$$

$$x = \frac{2\sqrt{x+1} + 2}{2} = \sqrt{x+1} + 1 \quad (\text{option c satisfy this})$$

$$\therefore \boxed{x=3}$$

$$x-1 = \sqrt{x+1}$$

squaring both sides

$$(x-1)^2 = x+1 \rightarrow x^2 - 2x + 1 = x + 1$$

$$x^2 = 3x \rightarrow \boxed{x=3}$$

OR if $y = \sqrt{x - 2\sqrt{x - 2\sqrt{x - \dots}}}$

$$x-y = 2 \rightarrow x-1 = 2 \rightarrow \boxed{x=3}$$

$$xy = x$$

$$\boxed{y=1}$$

119. Let $x = \sqrt{3 + 2\sqrt{3 - 2\sqrt{13 + 2\sqrt{3 - \dots}}}}$ then x equals

- 1) 1 2) 72 3) 2 4) None

$$\frac{\sqrt{4x^2 - 3 \cdot 2^2 + 2}}{2} \Rightarrow \frac{0+2}{2} \Rightarrow \boxed{1}$$

120. Let $x = \sqrt{10 + 3\sqrt{10 - 3\sqrt{10 + 3\sqrt{10 - \dots}}}}$; then x equals

- 1) $\frac{\sqrt{19} - 3}{2}$ 2) $3\sqrt{5}$ 3) $\frac{\sqrt{13} + 3}{2}$ 4) $\frac{\sqrt{17} + 1}{2}$

$$\frac{\sqrt{4x^2 - 3 \cdot 3^2 + 3}}{2} \Rightarrow \boxed{\frac{\sqrt{13} + 3}{2}}$$

121. $\sqrt{7 + 2\sqrt{7 - 2\sqrt{7 + 2\sqrt{7 - 2\sqrt{7 - \dots}}}}} = ?$

- 1) $\sqrt{15}$ 2) 4 3) $\frac{3 + \sqrt{15}}{2}$ 4) 3

$$x = \frac{\sqrt{4x^2 - 3 \cdot 2^2 + 2}}{2} \Rightarrow \frac{\sqrt{16} + 2}{2} \Rightarrow \frac{6}{2} \Rightarrow \boxed{3}$$

122. $x = \sqrt{7 - 2\sqrt{7 + 2\sqrt{7 - 2\sqrt{7 + \dots}}}}$, then x equals

$$x = \frac{\sqrt{4x^2 - 3 \cdot 2^2 - 2}}{2} \Rightarrow \frac{\sqrt{16} - 2}{2} \Rightarrow \frac{2}{2} \Rightarrow \boxed{1}$$

123. Find $\sqrt[3]{210 + \sqrt[3]{210 + \sqrt[3]{210 + \dots}}}$?

- 1) 5 2) 6 3) 6.5 4) 7

1 के diff में Break करो $\rightarrow \frac{210}{5 \times 6 \times 7}$ middle no. is Ans = $\boxed{6}$

OR $x = \sqrt[3]{210 + \sqrt[3]{210 + \sqrt[3]{210 + \dots}}}$ $\therefore x^3 - x = 210$ $\therefore (x-1)x(x+1) = 5 \times 6 \times 7$
 $x = \sqrt[3]{210+x}$ cube $x^3 = 210+x$ $x(x^2-1) = 210$

$$\boxed{x=6}$$

124. Let $x = \sqrt{7 - 2\sqrt{7 + 2\sqrt{7 - 2\sqrt{7 + \dots}}}}$; then x equals

- 1) 1 2) $\sqrt{2}$ 3) 2 4) none

$$\frac{\sqrt{4x^2 - 3 \cdot 2^2 - 2}}{2} \Rightarrow \frac{4-2}{2} \Rightarrow \boxed{1}$$



125. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest?

1) $2^{1/2}$

2) $3^{1/3}$

3) $4^{1/4}$

4) $12^{1/12}$

$$\text{LCM}(2, 3, 4, 6, 12) = 12$$

$$\therefore 2^6, 3^4, 4^3, 6^2, 12^1 \rightarrow \text{smallest} = 12^{\frac{1}{12}}$$

\downarrow
largest = $3^{\frac{1}{3}}$

126. The greatest number among 2^{72} , 5^{36} , 11^{24} and 3^{60} is

1) 2^{72}

2) 5^{36}

3) 11^{24}

4) 3^{60}

$$\text{LCM}(72, 36, 24, 60) = 12$$

\therefore divide all powers by 12

$$\therefore 2^6, 5^3, 11^2, 3^5 \rightarrow \text{largest} = \boxed{3^{60}}$$



127. The smallest among the numbers 7^{200} , 9^{150} , 6^{250} and 5^{300} is

1) 7^{200}

2) 5^{300}

3) 9^{150}

4) 6^{250}

divide all powers by 50.

$$7^4, 9^3, 6^5, 5^6 \quad \therefore \text{smallest} \rightarrow 9^3 \quad \therefore \boxed{9^{150}}$$

Largest $\rightarrow 5^6 \quad \therefore \boxed{5^{300}}$

128. The smallest in, $(\sqrt{8} + \sqrt{5})$, $(\sqrt{7} + \sqrt{6})$, $(\sqrt{10} + \sqrt{3})$ and $(\sqrt{11} + \sqrt{2})$ is:

1) $(\sqrt{8} + \sqrt{5})$

2) $(\sqrt{7} + \sqrt{6})$

3) $(\sqrt{10} + \sqrt{3})$

4) $(\sqrt{11} + \sqrt{2})$

sum = constant = 13

\therefore smallest = $\sqrt{11} + \sqrt{2}$ (diff max)

Largest = $\sqrt{7} + \sqrt{6}$ (diff min)

129. The smallest of $(\sqrt{23} + 2\sqrt{3})$, $(\sqrt{31} + 2)$, $(\sqrt{24} + \sqrt{11})$, $(\sqrt{29} + \sqrt{6})$ is:

1) $(\sqrt{23} + 2\sqrt{3})$

2) $(\sqrt{31} + 2)$

3) $(\sqrt{24} + \sqrt{11})$

4) $(\sqrt{29} + \sqrt{6})$

$$(\sqrt{23} + \sqrt{12}), (\sqrt{31} + \sqrt{4}), (\sqrt{29} + \sqrt{6}), (\sqrt{24} + \sqrt{11})$$

$\Rightarrow (\sqrt{23} + \sqrt{12}) \rightarrow$ Largest

$\Rightarrow (\sqrt{31} + \sqrt{4}) \rightarrow$ smallest

130. Which is the greatest among $(\sqrt{19} + \sqrt{31})$, $(\sqrt{23} + 3\sqrt{3})$, $(\sqrt{17} + \sqrt{33})$, 10?

(a) $(\sqrt{17} + \sqrt{33})$

(b) $(\sqrt{23} + 3\sqrt{3})$

(c) 10

(d) $(\sqrt{19} + \sqrt{31})$



$$(\sqrt{19} + \sqrt{31}), (\sqrt{23} + \sqrt{27}), (\sqrt{17} + \sqrt{33}), (\sqrt{25} + \sqrt{25})$$

\therefore Largest = 10 Smallest = $(\sqrt{17} + \sqrt{33})$

131. The smallest of $(\sqrt{69} + 2\sqrt{7}), (\sqrt{61} + 6), (5\sqrt{3} + 22)$ and $(\sqrt{58} + \sqrt{39})$ is:

- 1) $(\sqrt{61} + 6)$ 2) $(\sqrt{69} + 2\sqrt{7})$ 3) $(5\sqrt{3} + \sqrt{39})$ 4) $(\sqrt{58} + \sqrt{39})$

$$(\sqrt{69} + \sqrt{28}), (\sqrt{61} + \sqrt{36}), (\sqrt{55} + \sqrt{22}), (\sqrt{58} + \sqrt{39})$$

Smallest = $\sqrt{55} + \sqrt{22} \rightarrow 5\sqrt{3} + \sqrt{22}$

132. Which is the greatest among $(\sqrt{24} + \sqrt{10}), (\sqrt{30} + \sqrt{8}), (\sqrt{15} + 4), (\sqrt{12} + \sqrt{20})$?

- 1) $(\sqrt{24} + \sqrt{10})$ 2) $(\sqrt{30} + \sqrt{8})$ 3) $(\sqrt{15} + 4)$ 4) $(\sqrt{12} + \sqrt{20})$

$$(\sqrt{24} + \sqrt{10}), (\sqrt{30} + \sqrt{8}), (\sqrt{15} + \sqrt{16}), (\sqrt{12} + \sqrt{20})$$

Product = constant \therefore Sum max = Largest
Sum min = Smallest

\therefore Largest no. $\rightarrow \sqrt{30} + \sqrt{8}$



133. Which of the following statement(s) is/are TRUE?

- I. $\sqrt{11} + \sqrt{7} < \sqrt{10} + \sqrt{8}$ II. $\sqrt{17} + \sqrt{11} > \sqrt{15} + \sqrt{13}$
 1) Only I 2) Only II 3) Both I and II 4) Neither I nor II.

I (v) II (x) \therefore only (1) is true.

134. Which of the following relation(s) is/are TRUE?

- I. $(\sqrt{15} + \sqrt{7}) < (2\sqrt{22})$ II. $(\sqrt{17} + \sqrt{5}) < (\sqrt{20} + \sqrt{2})$
 1) Only I 2) Only II 3) Neither I nor II 4) Both I and II

$$\sqrt{15} + \sqrt{7} < \sqrt{22} + \sqrt{22} \quad (v) \quad \text{only (1) is true.}$$

$$\sqrt{17} + \sqrt{5} < \sqrt{20} + \sqrt{2} \quad (x)$$

135. Which is the greatest among

$$(\sqrt{15} - \sqrt{10}), (\sqrt{19} - \sqrt{6}), (\sqrt{18} - \sqrt{7}), (\sqrt{17} - \sqrt{8})?$$

- 1) $(\sqrt{15} - \sqrt{10})$ 2) $(\sqrt{19} - \sqrt{6})$
 3) $(\sqrt{18} - \sqrt{7})$ 4) $(\sqrt{17} - \sqrt{8})$



$\sqrt{15}, \sqrt{19}, \sqrt{18}, \sqrt{17} \rightarrow \text{Largest} = \sqrt{19}$

$\sqrt{10}, \sqrt{6}, \sqrt{7}, \sqrt{8} \rightarrow \text{smallest} = \sqrt{6}$

$\therefore \text{Largest - smallest} = \text{Largest} \quad \therefore \boxed{\sqrt{19} - \sqrt{6}}$

136. Which is the greatest among

$$(\sqrt{17} - \sqrt{14}), (\sqrt{19} - 4), (\sqrt{22} - \sqrt{19}), (\sqrt{13} - \sqrt{10})$$

- 1) $(\sqrt{17} - \sqrt{14})$ 2) $(\sqrt{19} - 4)$ 3) $(\sqrt{22} - \sqrt{19})$ 4) $(\sqrt{13} - \sqrt{10})$

$$\frac{3}{\sqrt{17} + \sqrt{14}}, \frac{3}{\sqrt{19} + \sqrt{14}}, \frac{3}{\sqrt{22} + \sqrt{19}}, \frac{3}{\sqrt{13} + \sqrt{10}}$$

$\therefore \text{smallest denominator} \rightarrow \sqrt{13} + \sqrt{10}$

$$\therefore \text{Greatest} = \boxed{\sqrt{13} - \sqrt{10}} \quad \text{OR} \quad \text{min sum} = 13 + 10 = 23 \\ \therefore \text{Largest} = \boxed{\sqrt{13} - \sqrt{10}}$$

137. Which one among the following is the smallest?

- 1) $\sqrt{201} - \sqrt{199}$ 2) $\sqrt{101} - \sqrt{99}$ 3) $\sqrt{301} - \sqrt{299}$ 4) $\sqrt{401} - \sqrt{399}$

$$\text{max sum} = 401 + 399 = 800$$

$$\therefore \text{smallest} = \boxed{\sqrt{401} - \sqrt{399}}$$

$$\text{min sum} = 101 + 99 = 200$$

$$\therefore \text{Largest} = \boxed{\sqrt{101} - \sqrt{99}}$$

138. Which of the following statement(s) is/are TRUE?

- I. $33^3 > 3^{33}$ II. $333 > (3^3)^3$
 1) Only I 2) Only II 3) Both I and II 4) Neither I nor II

$$\text{(I)} \rightarrow (3^3)^3 < (3^3)^3 \quad \therefore \text{I (x)}$$

$$\text{II} \rightarrow 333 < 27^3 \quad \therefore \text{II (x)}$$

$\therefore \text{Neither I nor II.}$

139. Which of the following is TRUE?

- I. $\sqrt[3]{11} > \sqrt{7} > \sqrt[4]{45}$ II. $\sqrt{7} > \sqrt[3]{11} > \sqrt[4]{45}$ III. $\sqrt{7} > \sqrt[4]{45} > \sqrt[3]{11}$ IV. $\sqrt[4]{45} > \sqrt{7} > \sqrt[3]{11}$

- 1) Only I 2) Only II 3) Only III 4) Only IV

$$\begin{aligned} & 11^{\frac{1}{3}}, 7^{\frac{1}{2}}, 45^{\frac{1}{4}} \quad \text{Lcm}(3, 2, 4) = 12 \\ & \rightarrow (11^{\frac{1}{3}})^{12}, (7^{\frac{1}{2}})^{12}, (45^{\frac{1}{4}})^{12} \quad \therefore 45^2 \times 7^2 \approx (45 \times 7)^2 \\ & \rightarrow 11^4, 7^6, 45^3 \quad \therefore (343)^2 > (315)^2 > (121)^2 \end{aligned}$$



$$\rightarrow (11^2)^2, (7^3)^2, 45^2 \times 45 \dots \sqrt{7} > \sqrt[4]{45} > \sqrt[3]{11}$$

$$\rightarrow (121)^2, (343)^2 \approx 315^2$$

140. Which of the following statement(s) is/are TRUE?

I. $(65)^{1/6} > (17)^{1/4} > (12)^{1/3}$

II. $(17)^{1/4} > (65)^{1/6} > (12)^{1/3}$

III. $(12)^{1/3} > (17)^{1/4} > (65)^{1/6}$

1) Only I

2) Only II

3) Only III

4) None of these

$$65^{\frac{1}{6}}, 17^{\frac{1}{4}}, 12^{\frac{1}{3}}$$

$$\rightarrow (65^{\frac{1}{6}})^{12}, (17^{\frac{1}{4}})^{12}, (12^{\frac{1}{3}})^{12}$$

$$\rightarrow 65^2, 17^3, 12^4 \Rightarrow \begin{array}{ccc} 65^2 & 17^3 & 144^2 \\ 65^2 & 17^2 \times 17 & 144^2 \\ 65^2 & 17^2 \times 4^2 & 144^2 \\ 65^2 & = 68^2 & 144^2 \end{array}$$

$$\therefore (12)^{\frac{1}{3}} > (17)^{\frac{1}{4}} > (65)^{\frac{1}{6}}$$



141. Which of the following is TRUE?

I. $\frac{1}{\sqrt[3]{12}} > \frac{1}{\sqrt[4]{29}} > \frac{1}{\sqrt{5}}$ II. $\frac{1}{\sqrt[4]{29}} > \frac{1}{\sqrt[3]{12}} > \frac{1}{\sqrt{5}}$ III. $\frac{1}{\sqrt{5}} > \frac{1}{\sqrt[3]{12}} > \frac{1}{\sqrt[4]{29}}$ IV. $\frac{1}{\sqrt{5}} > \frac{1}{\sqrt[4]{29}} > \frac{1}{\sqrt[3]{12}}$

1) Only I

2) Only II

3) Only III

4) Only IV

$$\sqrt[3]{12} \quad \sqrt[4]{29} \quad \sqrt{5}$$

Take powers = 12

$$\therefore \begin{array}{ccc} 12^4 & 29^3 & 5^6 \\ 144^2 & 29^2 \times 5^2 & 125^2 \\ 144^2 & 145^2 & 125^2 \end{array}$$

$$\boxed{\begin{array}{l} a > b > c \\ \frac{1}{a} < \frac{1}{b} < \frac{1}{c} \end{array}}$$

$$\therefore \frac{1}{\sqrt{5}} > \frac{1}{\sqrt[3]{12}} > \frac{1}{\sqrt[4]{29}}$$

142. While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then, the minimum possible value of the sum of squares of the other two numbers is-

1) 41

2) 40

3) 104

4) 36

$$a \times b \times 73 - a \times b \times 37 = 720 \quad \therefore \text{integer not given}$$

$$ab(36) = 720$$

$$ab = 20$$

$$\therefore ab = 20$$

$$\downarrow \quad \downarrow \\ \sqrt{20} \times \sqrt{20} = 20$$

$$\therefore (a^2 + b^2)_{\min} = 20 + 20 = \boxed{40}$$

143. Find the approximate value of these?

1) $\sqrt{11} = 3.28578$

2) $\sqrt{144} = 10.666$

3) $\sqrt[3]{56} = 3.783$

4) $\sqrt[4]{74} = 2.892$

a) $\sqrt{11}$
 $\rightarrow 3 + \frac{2}{7} \rightarrow 3.28$

b) $\sqrt{114}$
 $\rightarrow 10 + \frac{14^2}{2+3} \rightarrow 10.66$

c) $\sqrt[3]{56}$
 $\rightarrow 3 + \frac{29}{37} \rightarrow 3.78$

d) $\sqrt[4]{74}$
 $\rightarrow 2 + \frac{58}{65} \rightarrow 2.89$

144. The ascending order of $(2.89)^{0.5}$, $2-(0.5)^2$, $\sqrt{3}$ and $\sqrt[3]{0.008}$ is

1) $2-(0.5)^2, \sqrt{3}, \sqrt[3]{0.008}, (2.89)^{0.5}$

2) $\sqrt[3]{0.008}, (2.89)^{0.5}, 2-(0.5)^2, \sqrt{3}$

3) $\sqrt[3]{0.008}, (2.89)^{0.5}, \sqrt{3}, 2-(0.5)^2$

4) $\sqrt{3}, \sqrt[3]{0.008}, 2-(0.5)^2, (2.89)^{0.5}$

$$(2.89)^{0.5} = (1.7)^{2 \times 0.5} = (1.7)^1 = 1.7$$

$$2 - (0.5)^2 = 2 - 0.25 = 1.75$$

$$\sqrt{3} = 1.73 \quad \therefore \text{Ascending order} \rightarrow 0.2, 1.7, 1.73, 1.75$$

$$\sqrt[3]{0.008} = 0.2$$

145. Which of the following statement(s) is/are TRUE?

I. $\sqrt{121} + \sqrt{12321} + \sqrt{1234321} = 1233$

II. $\sqrt{0.64} + \sqrt{64} + \sqrt{36} + \sqrt{0.36} > 15$

1) Only I

2) Only II

3) Neither I or II

4) Both I and II

I $\rightarrow 11 + 111 + 1111 = 1233 (\checkmark)$

II $\rightarrow 0.8 + 8 + 6 + 0.6 = 15.4 > 15 (\checkmark)$

Both are correct

146. Find the value of x, if $16^{\sqrt{x}} + 63^{\sqrt{x}} = 65^{\sqrt{x}}$

1) 4

2) 3

3) 0

4) 2



$2 \sqrt{3}$ का root नहीं निकलेगा, Irrational no. से Add नहीं होगा
 ० नहीं होगा $\therefore 16^\circ + 63^\circ = 65^\circ$
 $1 + 1 = 1$ (Not possible)

\therefore only 4 is possible.

OR $16, 63, 65$ is a pythagoras triplet

$$16^2 + 63^2 = 65^2 \quad \therefore \sqrt{x} = 2 \rightarrow x = 4$$

147. Find the value of x, if $12^{\sqrt{x}} + 12^{\sqrt{x}} + 21^{\sqrt{x}} = 29^{\sqrt{x}}$

- 1) 4 2) 3 3) 16 4) 2

Put 4 and check $\rightarrow 12^2 + 16^2 + 21^2 = 29^2$
 $144 + 256 + 484 = 784 = 29^2$

$\therefore x = 4$

148. Solve $\sqrt{21 + \sqrt[3]{59 + \sqrt{16 + \sqrt[3]{722 + \sqrt{49}}}}} = ?$

- 1) 4 2) 5 3) 6 4) 7

$$\sqrt{21 + \sqrt[3]{59 + \sqrt{16 + \sqrt[3]{722 + \sqrt{49}}}}}$$

$\sqrt[3]{729} = 9$ ← solve from back side

$$\sqrt{16+9} = 5$$

$$\sqrt[3]{59+5} = 4$$

$$\sqrt{21+4} = \boxed{5} \text{ Ans}$$



149. $\left\{ \frac{\left[(1.331)^{-1} + (1.331)^{-2} + \dots + (1.331)^{-6} \right]}{\left[(1.331)^{-2} + (1.331)^{-3} + \dots + (1.331)^{-7} \right]} \right\}^{\frac{1}{3}}$

- 1) $\frac{221}{110}$ 2) $\frac{121}{100}$ 3) $\frac{241}{110}$ 4) $\frac{11}{10}$

$$\left\{ \frac{\left[(1.33)^{-1} + (1.33)^{-2} + \dots + (1.33)^{-6} \right]}{1.33^{-1} \left[(1.33)^{-1} + (1.33)^{-2} + \dots + (1.33)^{-6} \right]} \right\}^{-\frac{1}{3}} + 1.1$$

$$\Rightarrow \left(\frac{1}{1.33^{-1}} \right)^{-\frac{1}{3}} + 1.1 \Rightarrow \frac{1}{(1.1)^{3 \times \frac{1}{3}}} + 1.1 \Rightarrow \frac{1}{1.1} + 1.1 \Rightarrow \boxed{\frac{22}{110}}$$

150. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}, c^z = \sqrt{a}$, then find xyz = ?

- 1) $\frac{1}{6}$ 2) $\frac{1}{9}$ 3) $\frac{1}{24}$ 4) $\frac{1}{12}$

$$\begin{array}{l|l|l} a^x = \sqrt{b} & b^y = c^{\frac{1}{3}} & c^z = \sqrt{a} \\ b = a^{2x} & (a^{2x})^y = (a^{\frac{1}{2}z})^{\frac{1}{3}} & c = a^{\frac{1}{2}z} \\ a^{2xy} = a^{\frac{1}{2}z} & \therefore 2xy = \frac{1}{6z} & \boxed{xyz = \frac{1}{12}} \end{array}$$

151. $\sqrt{423 \times 424 \times 425 \times 426 + 1}$ is ?

- 1) Rational number 2) Irrational number 3) Rational integer 4) None

$\sqrt{423 \times 424 \times 425 \times 426 + 1} \rightarrow$ perfect square no.

\therefore Rational Integer

152. If $(x-2)a(x-5a)(x-8a)(x-11a) + ka^4$ is perfect square then k = ?

- 1) 49 2) 81 3) 64 4) 0

Common diff of 4 terms = $3a$

To make it perfect square $\rightarrow (3a)^4$ is added $\rightarrow 81a^4$

$$\therefore ka^4 = 81a^4 \quad \therefore \boxed{k=81}$$



153. If $x(x-1)(x-2)(x-3) + 1 = k^2$, then which one of the following is a possible expression for k?

- 1) $x^2 - 3x + 1$ 2) $x^2 - 3x - 1$ 3) $x^2 + 3x - 1$ 4) $x^2 - x - 1$

$$(I \times IV + d^2)^2 \rightarrow (x(x-3) + 1^2)^2 \rightarrow (x^2 - 3x + 1)^2 = k^2$$

$$\therefore k = \boxed{x^2 - 3x + 1}$$